STOKES MATRIX APPROACH TO CLUTTER MODELLING

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

by

P N SRIKANTH

to the

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
SEPTEMBER, 1984

6/9/84

CERTIFICATE

Certified that the work reported in the thesis titled "STOKES MATRIX APPROACH TO CLUTTER MODELLING" has been carried out under our supervision by Mr P.N.Srikanth and has not been submitted elsewhere for a degree.

Dr P.R.K.Rao
Professor
Dept of Elect Engg
IIT Kanpur

Dr K.R.Sarma
Professor
Dept of Elect Engg.
IIT Kanpur

ACKNOWLEDGEMENTS

I am greatly indebted to my guides Dr. K.R. Sarma and Dr. P.R.K. Rao for their invaluable guidance and constant encouragement extended to me during my thesis work.

I also thank Dr. N.C. Mathur and Dr. V.P. Sinha for the help in solving some of the problems I faced in this work.

I thank all of my friends who made my stay at I.I.T. Kanpur enjoyable.

– P.N. SRIKANTH

A B S T R A C T

This thesis is an attempt to model the electromagnetic returns from clutter-producing targets that degrade the Radar detection performance.

The study is carried out in the frame-work of scattering theories employing target and Stokes reflection matrices. The use of Stokes reflection matrix allows for the decomposition of a target into a deterministic target and a noise target. The deterministic part of the target model admits a characterization of a target in terms of five phenomenological parameters orientation, helicity, skip and characterstic angles and 'magnitude' of the target.

Expressions for target scattering and Stokes reflection matrices for static surfaces modelled as random rough surfaces are derived. These are extended to obtain expressions for doppler spectrum of the scattered electromagnetic returns when temporal variations of the surfaces are present.

To illustrate these ideas, an asphalt road and a sea-surface are taken as examples. The variation of five phenomenological parameters associated with above two targets with frequency, aspect angle and surface roughness are also studied.

CONTENTS

		Page
CHAPTER 1	INTRODUCTION	1
1.1	Motivation and background	1
1.2	Organization of the thesis	5
CHAPTER 2:	POLARIZATION, TARGET SCATTERING AND STOKES REFLECTION MATRICES	7
2.1	Polarization and its representation	7
2.2	Target Scattering Matrix	16
2.3	Stokes Reflection Matrix	23
CHAPTER 3	SCATTERING FROM RANDOM ROUGH SURFACES	30
3.1	Review of Scattering theories	30
3.2	Derivation of mean and variance of TSM	35
3.3	Shadow function	41
3.4	Effect of temporal variation of rough surface	43
CHAPTER 4:	APPLICATION TO PRACTICAL SURFACES	49
4.1	Application to asphalt road	49
4.2	Application to a sea-surface	50
	APPENDIX - A	55
CHAPTER 5:	CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK	56
5.1	Conclusion	56
5.2	Suggestions for further work	56
	REFERENCES	57

CHAPTER 1

INTRODUCTION

1.1 Motivation and background

The received signal in radar due to scattering of electromagnetic waves from objects other than the desired one is termed as clutter. For better performance of the radar receiver, the effects due to clutter must be minimized. In order to minimize these effects, one must know the characterstics of the clutter. The experimental data about the clutter characterstics allow one to design radar receivers. But these data are non-stationary and varies with geographical and radar parameters. Hence it is desirable to have a theoritical model for clutter. This gives insight into the clutter phenomena, in addition to the design of adaptive radar receivers.

A number of clutter models are available in literature. One such model considers the target as a collection of ellipsoids with varying cross-sections[5]. The scattered power return due to a single ellipsoid was calculated first and the total scattered power is computed as the some of all the contributions from individual ellipsoids. But such an approach is fundamenta

incorrect, as the presence of other ellipsoids will effect the scattering characterstics of individual ellipsoids. Wong et.al [4] modelled the clutter as a random collection of rotating dipole scatterers. The rotation of dipoles represents the random motion of the objects such as leaves, vegetation, branches etc. due to wind forces. But the expressions obtained using this model for doppler spectrum is not related to any of the physical characterstics of the surface like its shape, roughness or permittivity. It is normal practice in detection and estimation theory to assume some kind of clutter distribution like Gaussian, log-normal or weibull for the design of Radar receivers [6]. In these, some parameters are introduced whose values are obtained by fitting with experimental data. But these parameters are also not related to any of the properties of the surface. Hence it is desirable to obtain a model which contains parameters which are related to the physical properties of the surface.

The characterstics of clutter can also be studied from its target scattering matrix (TSM) and Stokes reflection matrix associated with it. These two matrices, express the scattering and polarization properties of the target. Thus the information content in the polarization which was neglected in the past is utilized

to study the clutter characterstics. The study of TSM and Stokes reflection matrix reveals some interesting properties of the target. The eigenvalues and eigen-vectors associated with the TSM is related to some of the properties of TSM through five phenomenological parameters proposed by Huynen [1]. The basic idea is to decomposed a target return into one due to a deter-ministic part of the target and a noisy part. The TSM corresponding to the deterministic target is obtained and the associated five parameters represent clutter characterstics like its symmetry, non-symmetry, size and orientation.

TSM and Stokes reflection matrices can be obtained experimentally. Huynen has suggested an experimental procedure to measure the TSM[3] using dual polarized antennas. For details of experimental procedure, we refer to the above cited paper by Huynen. Theoretically, these matrices can be obtained theoretically utilizing scattering theories. Scattering theories are well developed in the literature. In fact, several excellent books are devoted to this topic. This includes the book by Beckmann and Spizzichino [7] and Bass and Fuks [9]. These scattering theories are utilized to model clutter especially sea clutter[17]. Apart from clutter modelling

scattering theories also find application in the area of remote sensing [22].

In this thesis, we compute the scattering matrix and Stokes reflection matrix associated with a surface modelled as random rough surface. The Stokes reflection matrix thus obtained is decomposed into one due to the deterministic part of the target and other due to the noisy part of the target. The TSM associated with deterministic part is obtained and the five phenomenological parameters are calculated from TSM.

1.2 ORGANIZATION OF THE THESIS:

In chapter 2, we review some of the concepts of polarization and its representation. The properties of TSM and Stokes reflection matrices are discussed in detail.

In chapter 3, we obtain expressions for mean and variance of TSM for a static surface modelled as a random rough surface. These expressions are then extended to include the temporal variations of the surface.

In chapter 4, we numerically compute the Stokes reflection matrices for an asphalt road and sea-surface. The TSM corresponding to deterministic part of the target is obtained and the associated target parameters are computed. The variation of these parameters with aspect angle, a frequency and surface roughness are also studied.

The results obtained in chapters 2,3,and 4 are discussed and concluded upon in chapter 5. Suggestions are given for the further improvements of the present model.

*****

CHAPTER 2

POLARIZATION TARGET SCATTERING AND STOKES REFLECTION MATRICES

In this Chapter we discuss the concept of polarization and its representation. This is followed by the discussion of some properties of target scattering matrices (TSM) and their relationship with five phenomenological parameters and stokes reflection matrices.

2.1 Polarization and its representation

Let E and H denote the electric and magnetic fields of a electromagnetic (e.m.) wave at a point in a medium with associated poynting vector S in the Z-directon of the rectangular co-ordinate system XYZ. Then the E and H fields lie in the transversal plane x-y. Let x and y components of electric field E be

$$E_x = E_{xo}\text{Cos}\,(wt + \theta_1) \quad ; \quad E_y = E_{yo}\text{Cos}\,(wt + \theta_2) \qquad \ldots(2.1.1)$$

where $\theta_1 = \phi_x - kz$ and $\theta_2 = \phi_y - kz$ with k describing the wavenumber associated with the medium

Denoting the phase-difference ($\theta_1 - \theta_2$) bet' E_x and E_y components by δ , the following can be shown to hold good.

$$\frac{Ex^2}{Ex_o^2} - \frac{2ExEy}{E_{xo}E_{yo}} \cos\delta + \frac{E_y^2}{E_{yo}^2} = \sin^2\delta$$

$$\text{or} \quad aE_x^2 - bE_xE_y + CE_y^2 = 1$$

$$\text{where } a = \frac{1}{E_{xo}^2 \sin^2\delta} \; ; \; b = \frac{2\cos\delta}{E_{xo}E_{yo}\sin^2\delta}, \; c = \frac{1}{E_{yo}^2 \sin^2\delta}$$

The ellipse specified by eqn (2.1.2) is called polarization ellipse and is described by following three parameters.

i) ellipticity angle $\tau = \tan^{-1}\left(\frac{OA}{OB}\right)$ $[-45^o \leq \tau \leq 45^o]$

ii) magnitude of the ellipse $m = \{OA^2+OB^2\}^{\frac{1}{2}} = AB$ ... (2.1.3)

iii) orientation angle of the ellipse $= \psi$ (see Fig. 2.1.1.)

These three parameters m, ψ and τ completely describes the polarization ellipse and are related to E_1, E_2, δ as under:

$$m = \{E_1^2 + E_2^2\}^{\frac{1}{2}}$$

$$\tan 2\psi = \frac{2E_1E_2}{E_1^2-E_2^2} \cos\delta \qquad \text{...(2.1.4)}$$

$$\sin 2\tau = \frac{2E_1E_2}{E_1^2+E_2^2} \sin\delta$$

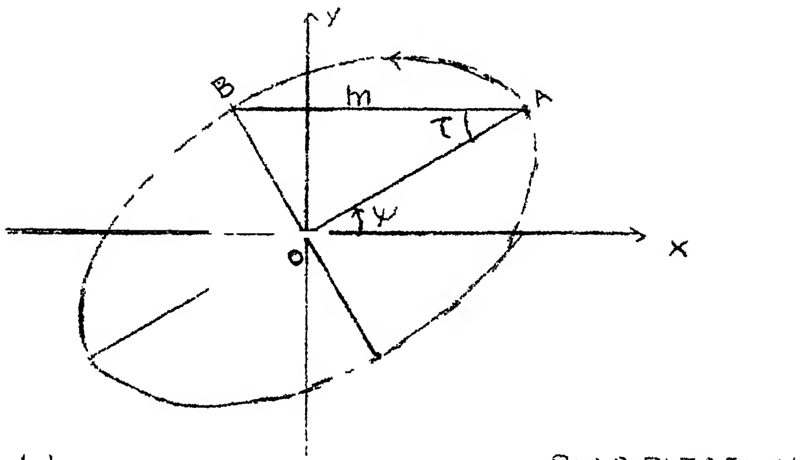
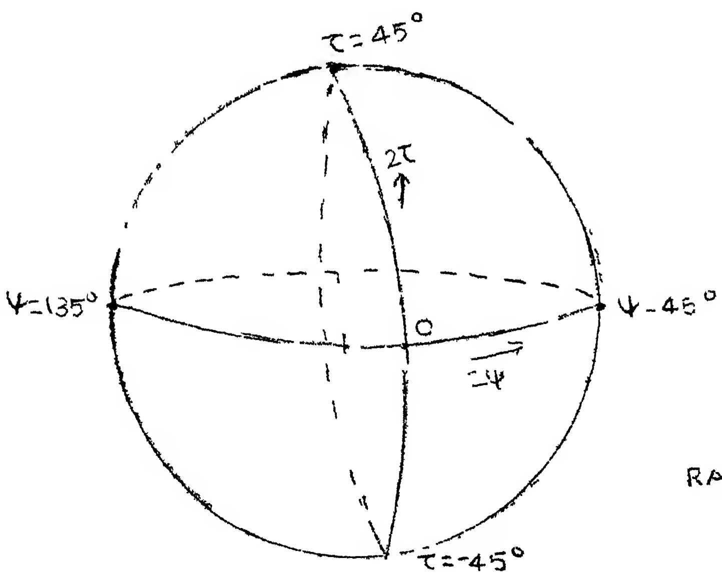


Fig 2 1 1

RIGHT SENSED POLARIZATION ELLIPSE IN A FIXED PLANE



RADIUS OF THE SPHERE = m

Fig 2 2 2 POINCARE SPHERE

It may be noted that m^2 is a measure of the total power in the e.m. wave.

For the purposes of this study, instead of the representation for the electric field given in (2.1.1), an alternative representation in terms of m. and is more convenient. We develop this alternative representation for the case of zero orientation angle first and then for arbitary angle

For $\delta = \pi/2$, we have $\psi = o^\circ$

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_1 \cos(\omega t + \theta_1) \\ E_2 \cos(\omega t + \theta_2) \end{bmatrix} = \mathrm{Re} \begin{bmatrix} E_1 e^{(j\omega t + \theta_1)} \\ E_2 e^{(j\omega t + \theta_2)} \end{bmatrix}$$

$$= \mathrm{Re} \begin{bmatrix} E_1 \\ E_2 \ e^{j\delta} \end{bmatrix} e^{(j\omega t + \theta_1)} = \mathrm{Re} \begin{bmatrix} E_1 \\ +jE_2 \end{bmatrix} e^{(j\omega t + \theta_1)}$$

Substituting $E_1 = m\cos\tau$; $E_2 = -m\sin\tau$, we get in complex notation,

$$\underline{E} = m \begin{bmatrix} \cos\tau \\ -j\sin\tau \end{bmatrix} e^{j\theta_1}$$

where following usual practice we have supressed exp (jwt) term.

The representation for a arbitary orientation angle corresponds to a rotation of the x-y co-ordinates thro' an angle ψ . And $\underline{E}$ may be seen to be given by

$$\underline{E}\ (a,\psi,\tau) = m\begin{bmatrix}\cos\psi & -\sin\psi\\ \sin\psi & \cos\psi\end{bmatrix}\begin{bmatrix}\cos\tau\\ j\sin\tau\end{bmatrix} e^{j\theta_1} \qquad ..(2.1.5)$$

where θ_1 is the absolute phase.

The above representation is often called the geometrical representation.

Some of the special cases of interest are -

If $E_1=0$, the wave is linearly polarized in x - direction.
If $E_2=0$, the wave is linearly polarized in y - direction
If $\delta=0^\circ$ $E_1=E_2$, the wave is linearly polarized with 45° orientation.
If $\delta=\pm 90^\circ$, and $E_1=E_2$ the wave is circularly polarized.

$\delta = +90^\circ$ represents left circularly polarized wave and $\delta = -90^\circ$ represents right circularly polarized wave. It may be noted that our convention about left and right circular polarizations are in accordance with IEEE standards is at variance with the practice in conventional optics.

The geometrical parameters m, ψ, and τ can be represented on a sphere called Poincare sphere [1]. In this sphere each polarization state is represented by a unique point. The size of the sphere depends on the magnitude m of the ellipse and the latitude and longitude of a point represent 2ψ and 2τ associated with $\underline{E}$ at that point. The following

situations are of special interest

(1) The equator of the Poincare sphere represents all linearly polarized waves ($2\tau = 0^0$)

(2) The poles represent circularly polarized waves ($2\psi = \pm 90^0$).

(3) The upper hemisphere represents all left sensed polarizations and lower hemisphere represents all right sensed polarizations.

We may also associate stokes vector with polarization[12] (   ). The stokes vector $\underline{g}$ is defined as

$$\underline{g} = (I, Q, U, V)^T$$

where
$$I = \frac{E_1^2}{Z_1} + \frac{E_2^2}{Z_2}$$

$$Q = \frac{E_1^2}{Z} - \frac{E_2^2}{Z} \qquad \ldots \quad (2.1.6)$$

$$U = \frac{2}{Z} E_1 E_2 \cos\delta$$

$$V = \frac{2}{Z} E_1 E_2 \sin\delta$$

where Z is the intrinsic impedence of the medium.

In terms of geometrical parameters, these components of $\underline{g}$ are given by

$$I = m^2 \quad : \quad Q = m^2 \cos 2\psi \cos 2\tau$$

$$U = m^2 \sin 2\psi \cos 2\tau \; ; \; V = m^2 \sin^2 2\tau \qquad (2.1.7)$$

From the above relations, it can be seen that,

$$I^2 = Q^2 + U^2 \mp V^2 \qquad (2.1.8)$$

Thus only 3 of the 4 parameters are independent.
An electromagnetic wave satisfying the above condition is said to be completely polarized. An e.m. wave which does not satisfy above condition is said to be partially polarized or unpolarized.

In above discussions, E_1, E_2, δ are taken as constants. Let us now take them as random samples of a stationary, erogodic, random process. In this situation, the equatins (2.1.1. - 2.1.3) are modified as follows

$$Ex = E_{xo}(t) \cos (wt + \theta_1 (t));$$

$$E_{yo} = E_{yo}(t) \cos (wt + \theta_1 (t)) + \delta (t))$$

and the equation of ellipse becomes, and the equation of the ellipse becomes,

$$A(t) E_x^2 - b(t) E_x E_y + c(t) E_y^2 = 1 \qquad ..(2.1.8)$$

where $a(t) = \dfrac{1}{E_1^2(t) \sin^2 \delta (t)}$; $b(t) = \dfrac{2 \cos \delta (t)}{E_1(t) E_2(t) \sin^2 \delta (t)}$;

$$c(t) = \frac{1}{E_2^2 (t) \sin^2 \delta (t)}$$

Thus the major and minor axis in this case are time varying. It corresponds to a wave whose orientation, eccentricity and size of the polarization ellipse continuously changes with time. The Stokes parameters for this case are given by the following time-averages ,

$$I = \frac{\langle E_1^2(t)\rangle}{Z} + \frac{\langle E_2^2(t)\rangle}{Z}$$

$$Q = \frac{\langle E_1^2(t)\rangle}{Z} - \frac{\langle E_2^2(t)\rangle}{Z}$$

$$U = \frac{2}{Z} \langle E_1(t)E_2(t) \cos\delta(t)\rangle \qquad \ldots \quad (2.1.9)$$

$$V = \frac{2}{Z} \langle E_1(t)E_2(t) \sin\delta(t)\rangle$$

where $\langle \quad \rangle$ indicates time averaging.

It may be shown that -

$I^2 \geqslant Q^2+U^2+V^2$ and e.m. waves satisfying this inequality strictly are called partially polarized provided all the quantities Q,U,V are not simultaneously equal to zero.

For unpolarized waves, $E_1(t)$ and $E_2(t)$ are uncorrelated and hence $U=V=0$. Since $Q = m \cos 2\psi \cos 2\tau$ and average value of $\langle \cos 2\psi \cos 2\tau \rangle = 0$, $Q=0$, Hence $Q=U=V=0$ for a unpolarized wave.

Thus the Q,U,V components represent the polarized partof the wave. If anyone of them is non-zero, it indicates that the wave contains a polarized component.

The degree of polarization is defined as the ratio of polarized power to total power. Thus

$$\text{degree of polarization} = \frac{\text{polarized power}}{\text{total power}}$$

$$\text{i.e.} \qquad d = \frac{(Q^2+U^2+V^2)^{\frac{1}{2}}}{I} \qquad \ldots (2.1.10)$$

The auto correlation and cross correlation of $E_1(t)$ and $E_2(t)$ are often used to define the complex correlation matrix (a) so called coherence matrix J [13] as follows:

$$\underline{J} = \begin{matrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{matrix} = \begin{matrix} \langle E_1(t)E_1^*(t)\rangle & \langle E_1(t)E_2^*(t)\rangle \\ \langle E_1^*(t)E_2(t)\rangle & \langle E_2(t)E_2^*(t)\rangle \end{matrix} \qquad --(2.1.11)$$

The complex degree of coherence Uxy is defined as follows

$$\mu_{xy} = \left(\frac{J_{xy}}{(J_{xx})^{\frac{1}{2}}(J_{yy})^{\frac{1}{2}}}\right) = (\mu_{xy})\, e^{J\theta_{xy}} \qquad ..(2.1.12)$$

where $\mu_{xy} \leq 1$

when $\mu_{xy} = 1$, the wave is said to be completely polarized.

when $\mu_{xy} \nleq 1$, the wave is said to be partially polarized.

when $\mu_{xy} = 0$, the wave is said to be unpolarized.

Thus the concept of temporal coherence and polarization are intimately connected.

Any partially polarized wave with Stokes vector $[I, Q, U, V]^T$ may be considered to consist of an unpolarized wave with an intensity $I_u^2 = I^2 - (Q^2 + U^2 + V^2)$ and a completely polarized wave with an intensity $I_p^2 = Q^2 + U^2 + V^2$. Thus $I^2 = I_u^2 + I_p^2$ is the sum of intensities of the polarized and unpolarized waves, and $I^2 \geq Q^2 + U^2 + V^2$ with equality sign holding good for completely polarized wave. This kind of decomposition of a partially polarized wave into a polarized wave and an unpolarized wave is utilized in the target decomposition theorems derived by Huyhen[1].

We conclude this discussion by listing the stokes parameters of some commonly encountered polarizations.-

1 linearly polarized wave with $E_2=0$ i.e.,

$$I = P, \quad Q=P; \; U=V=0$$

2 linearly polarized wave with $E_1=0$; (ie)

$$I = P, \quad Q=P, \; U=V=0$$

3 left circularly polarized wave with $E_1=E_2$, $=90^\circ$

$$I = P; \quad Q=0, \; U=0, \; V=P,$$

Where in above eqns P denotes power density.

2.2 Target Scattering Matrix

Let us consider an e.m. wave $\underline{E}^i$ incident on a point target. The scattered wave $\underline{E}^s$ from that point is given by

$$\underline{E}^s = S\underline{E}^i \qquad \text{.......(2.2.1)}$$

where S is called scattering matrix and is a 2x2 complex matrix.

$$\text{Let } S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \qquad \text{.......(2.2.2)}$$

where $s_{11} = \left.\dfrac{E_1^s}{E_1^i}\right|_{E_2^i = 0}$; $s_{12} = \left.\dfrac{E_1^s}{E_2^i}\right|_{E_1^i = 0}$

$s_{21} = \left.\dfrac{E_2^s}{E_1^i}\right|_{E_2^i=0}$, $s_{22} = \left.\dfrac{E_2^s}{E_2^i}\right|_{E_1^i = 0}$

In our work, we shall consider only targets with 2x2 complex symmetric matrics (i.e.) $s_{12} = s_{21}$.

In our work, we consider the orthognal, Horizental and vertical Polarizations pair (H-V) to define the scattering matrix S. This matrix S can easily be converted to any other matrix S_1 associated with some other orthogonal polarization pair base by simple transformations[3].

The polarization scattering properties of a target are contained in its TSM. The properties of this matrix, can be investigated conveniently using its eigenvalues and eigen vectors. Let us formulate eigen-value problem as follows :

$$S\,\underline{x} = t\,\underline{x}^* \qquad \ldots (2.2.5)$$

This equation has two solutions. Denoting by t_1, t_2 the eigenvalues and by $\underline{x1}$, $\underline{x2}$, the eigenvectors of (2.2.5) we have,

$$S\,\underline{x1} = t_1 t_1^* \quad \text{and } S\,\underline{x2} = t_2\,\underline{x2}^* \qquad \ldots (2.2.6)$$

The emgen vectors $\underline{x1}$ and $\underline{x2}$ are orthogonal and are normalized.

The method of computing these eigenvalues and emgen-vectors are slightly different from computing the conventional eigenvalues and eigenvectors. In general, direct solution of eqn. (2.2.5) is difficult. Hence we have to compute these values in a slightly different way, which we will discuss in

The scattered field   is given by - $\underline{E}^s = S\underline{E}^i$ where $\underline{E}^i$ is the incident field.

The scattered power Ps is given by -

$$Ps = \underline{E}^{s\,+}\,\underline{E}^{s} = (S\underline{E}^{1})^{+}(S\underline{E}^{1}) \qquad \ldots(2.2.7)$$

expressing $\underline{E}^{1}$ interms of $\underline{x1}$ and $\underline{x2}$,

$$\underline{E}^{1} = E_1^1\,\underline{x1} + E_2^1\,\underline{x2} \qquad \ldots(2.2.8)$$

substitution in eqn. (2.2.8) gives,

$$Ps = (SE_1^1\,\underline{x1} + SE_2^1\,x2)_{,} + (SE_1^1\,\underline{x1} + SE_2^1\,x2)$$

knowing that $S\underline{x1} = t\underline{x1}^*$ and $S\underline{x2} = t\underline{x2}^*$ and using the property that $\underline{x1}$ and $\underline{x2}$ are orthonormal vectors, pwe get,

$$P_s = |E_1^1|^2\,|t_1|^2 + E_2^1\;t_2^{\;2}$$

$$= p_A^2\,|t_1|^2 - |E_2^1|^2\,(|t_1|^2 - |t_2|^2) \qquad \ldots(2.2.9)$$

where $p_A^2 = |E_1^1|^2 + |E_2^1|^2 = g_A =$ total transmitted power.

Without loss of generality, we assume $|t_1| \geq |t_2|$ and let g_A be fixed.

It can be easily seen that eqn. (2.2.9) is maximum when $|E_2^1| = 0$

Thus $P_{max} = g_A|t_1|^2$. The value of $|t_1|$ is assigned as a parameter m which is called target magnitude. It may be viewed as a overall measure of the target size.

From above discussions, it can be seen that the principal eigen vector $\underline{x1}$ corresponds to polarization for which the return power is maximum. $\underline{x1}$ can be represented

in terms of geometrical parameters associated with it as follows -

$$\underline{x}_1 = \begin{bmatrix} \cos\psi_m & -\sin\psi_m \\ \sin\psi_m & \cos\psi_m \end{bmatrix} \begin{bmatrix} \cos\tau_m \\ j\sin\tau_m \end{bmatrix} \quad ..(2.2.11)$$

The ψ_m and τ_m associated with this max. polarization is assigned to the target and are called target orientation and target helicity respectively. The eigen values are in general, complex and can be expressed in the following form .-

$$t_1 = m\, e^{j\theta_1} \quad \text{and} \quad t_2 = \alpha m\, e^{j\theta_2} \quad .. (2.2.12)$$

where the conditions $|t_1| = m$ and $|t_1| \geqslant |t_2|$ if $\alpha \leqslant 1$ are satisfied.

Having found eigenvalues and eigenvectors, let us see about nulls associated with T.S.M. There are two kinds of nulls - Co-POL nulls and X-POL nulls. They are defined as follows

The Co-POL nulls is polarization of radar $\underline{n}$ of identical Tx and Rx antennas which produces zero voltage reception at receiver terminals.

The X-POL null is that polarization $\underline{n}$, of orthogonal TX and RX antennas, which produces zero voltage reception at the receiver terminals.

Mathematically, $\underline{n}' s\underline{n} = 0$ for CO-POL nulls

$\underline{n}_{\perp}' s\underline{n} = 0$ for X-POL nulls ...(2.2.12)

where $\underline{n}$, $n_{\perp}$ are orthogonal polarizations.

CO-POL and X-POL nulls are practically measurable quantities. Hence it is desireable to express these nulls in terms of eigen values and eigenvectors of T.S.M. Expressing polarization $\underline{n}$ interms of $\underline{x1}$ & $\underline{x2}$.

$$\text{Let } \underline{n} = n_1 \underline{x1} + n_2 \underline{x2} \qquad \ldots(2.2.13)$$

Substituting in equation for Co-POL nulls,

$$(n_1 \, x1' + n_2 x2') \quad (Sn_1 \underline{x1} + Sn_2 \underline{x2}) = 0$$

using $S\underline{x} = t\underline{x}^*$ and orthonormal properties of $\mathbf{x1}$ and $\underline{\mathbf{x}2}$, one finally gets,

$$\underline{n}_1 = -j(\alpha)^{\frac{1}{2}} e^{j(\Theta_2-\Theta_1)/2} \underline{x}_1 + \underline{x}_2$$

$$\underline{n}_2 = +j(\alpha)^{\frac{1}{2}} e^{j(\Theta_2-\Theta_1)/2} \underline{x}_1 + \underline{x}_2 \qquad \ldots(2.2.14)$$

expressing in symmetrical form -

$$\underline{n}_1 = (\alpha)^{\frac{1}{2}} e^{-j45^\circ} e^{j(\Theta_2-\Theta_1)/4} \underline{x}_1 + e^{j45^\circ} e^{-j(\Theta_2-\Theta_1)/4} \underline{x}_2$$

$$\underline{n}_2 = -(\alpha)^{\frac{1}{2}} e^{-j45^\circ} e^{j(\Theta_2-\Theta_1)/4} \underline{x}_1 + e^{j45^\circ} e^{-j(\Theta_2-\Theta_1)/4} \underline{x}_2$$

At this stage, for convenience, let us say $\alpha = \tan^2\gamma$, and

Thus $\underline{n_1}$ and $\underline{n_2}$ becomes,

$$\underline{n_1} = \sin\gamma \, e^{-j(45^{\circ}+\nu)} \underline{x}_1 + \cos\gamma \, e^{j(45^{\circ}+\nu)} \underline{x}_2$$

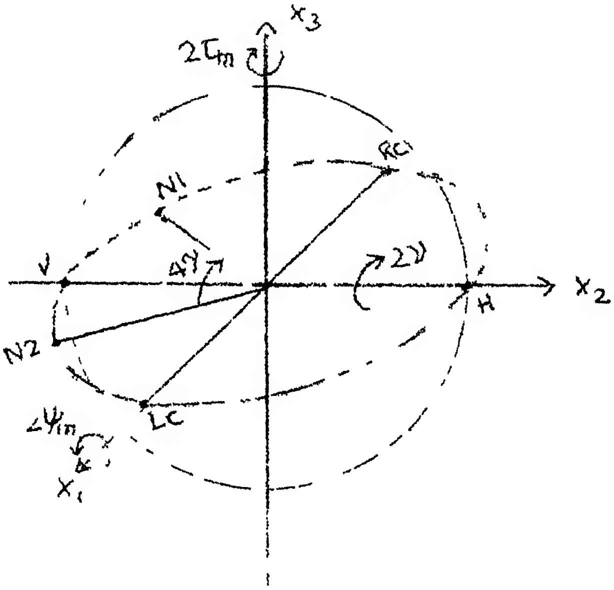
$$\underline{n_2} = -\sin\gamma \, e^{-j(45^{\circ}+\nu)} \underline{x}_1 + \cos\gamma \, e^{j(45^{\circ}+\nu)} \underline{x}_2 \quad \ldots(2.2.15)$$

where γ is called characterstic angle. As can be seen, the angle subtended by $\underline{n1}$, $\underline{n2}$ at the centre of the poincare' sphere is 4 γ. Hence parameter γ has acquired physical significance. The parameter ν is called "target skip angle" and we will discuss the role of this parameter later.

The X-POLS nulls can be easily seen to be eigen-vectors $\underline{x1}$ and $\underline{x2}$ themselves.

It is interesting to note that the polarizations $\underline{n1}$, $\underline{n2}$, $\underline{x1}$, $\underline{x2}$ when plotted in the Poincare' sphere lie on one great circle. The line joining $\underline{x1}$ and $\underline{x2}$ bisects the angle subtended by $\underline{n1}$ and $\underline{n2}$, when these points are connected to the Centre of the circle, they form the shape of a fork which is called Huynen's fork. From eqns (2.2.15), one can see that the parameter determines the rotation the 'prongs' of the fork about line joining $\underline{x1}$, $\underline{x2}$ by an amount of 2ν .

Given the CO-POL and X-POL nulls, one can determine TSM except for target magnitude and absolute phase ρ.



POLARIZATION NULLS

FIG 2.5.1

Returning to the eigen-values, we can write t_1 and t_2 with new values of α and $(\theta_2-\theta_1)$ as $t_1 = me^{2j(\nu+\rho)}$ and $t_2 = m\tan^2\gamma\ e^{-2j(\nu-\rho)}$ ...(2.2.16)

where is called the absolute phase, which dis-appears with power measurements.

Thus the six parameters $m, \psi_m, \tau_m, \gamma, \nu, \rho$ completely determines the TSM. Let us know express TSM in terms of these parameters or equivalently in terms of eigen-values and eigen-vectors.

We can construct the unitary transformation -

$$U = \begin{bmatrix} x1^+x1 & x2^+x1 \\ x1^+x2 & x2^+x2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

where orthonormal properties of <u>x1</u> and <u>x2</u> are used.

Thus $U^* = (U')^{-1}$ (or) $U^{-1} = U^{*\prime} = U^+$ ...(2.2.17)

we can use the U to bring S into diagonal form

$$U' SU = S_d \quad \text{where } S_d = \begin{bmatrix} t_1 & 0 \\ 0 & t_2 \end{bmatrix} \qquad \ldots(2.2.18)$$

Applying inverse transformation, noting that $U^{-1} = U^+$, we get

$$S = U^* S_d U^+ \qquad \ldots(2.2.19)$$

expressing U and S_d in terms of eigen-values and eigen-vectors, we finally get,

$$S = \begin{bmatrix} \cos\psi_m & -\sin\psi_m \\ \sin\psi_m & \cos\psi_m \end{bmatrix} \begin{bmatrix} \cos\tau_m & -\sin\tau_m \\ -j\sin\tau_m & \cos\tau_m \end{bmatrix} \begin{bmatrix} me^{2j(\nu+\rho)} & 0 \\ 0 & m\tan^2\gamma e^{-2j(\nu-\rho)} \end{bmatrix}$$

$$\cdot \begin{bmatrix} \cos\Psi_m & -\ j\sin\Psi_m \\ -j\sin\Psi_m & \cos\Psi_m \end{bmatrix} \begin{bmatrix} \cos\tau_m & \sin\Psi_m \\ -\sin\tau_m & \cos\Psi_m \end{bmatrix} \quad \ldots(2.2.20)$$

2.3 Stokes Reflection Matrix

Let a e.m. wave with stokes vector $\underline{Pi}\ [I^i, Q^i, U^i, V^i]^T$ is incident on a target.

Let the scattered wave have a stokes vector $\underline{Ps}\ [I^s, Q^s, U^s, V^s]^T$. Then these two vectors are related by -

$$\underline{Ps} = \begin{bmatrix} I^s \\ Q^s \\ U^s \\ V^s \end{bmatrix} \begin{bmatrix} m11 & m12 & m13 & m14 \\ m21 & M22 & m23 & m24 \\ m31 & m32 & m33 & m34 \\ m41 & m42 & m43 & m44 \end{bmatrix} \begin{bmatrix} I^i \\ Q^i \\ U^i \\ V^i \end{bmatrix} = R\ \underline{Pi} \quad (2.3.1)$$

Where R is called stokes reflection matrix (also called Mueller matrix).

Using law of reciprocity, perrin [14]) showedthat the elements $m_{ij} = m_{ji}$ and hence the matrix R is symmetrical. R can further be rewritten in the following form- Huyren,1970)

$$R = \begin{bmatrix} Ao+Bo & F & C & H \\ F & -Ao+Bo & G & D \\ C & G & Ao+B & -E \\ H & D & -E & Ao-B \end{bmatrix} \quad ..(2.3.2)$$

Rewriting that for a completely polarized wave,

$$I^{i^2} = Q^{i^2} + U^{i^2} + V^{i^2};$$

and for a partially polarized wave, $I^{s^2} > Q^{s^2} + U^{s^2} + V^{s^2}$, Huynen [1] arrives at the following conditions on the elements of R - matrix.

$$\begin{aligned} 2Ao(Bo+B) &\geqslant C^2 + D^2 \\ 2Ao(Bo-B) &\geqslant G^2 + H^2 \\ 2AoE &\geqslant CH - GD \\ 2AoF &\geqslant CG + DH \end{aligned} \tag{2 3 3}$$

For a target, which scatters a completely polarized wave, the above relations satisfy with equality sign. In this, case, the target is called a single target and the corresponding stokes reflection matrix is denoted by M. On the other hand, if a target scatters a partially polarized wave, the above relations donot satisfy with equality sign. In this case the target is called a distributed target and corresponding stokes reflection matrix is denoted by R.

For any arbitary targets, the stokes reflection matrix R associated with it can be decomposed into a single target M corresponding to complectely polarized part and a noise target corresponding to unpolarized part. This theorem called target decomposition theorem, follows from the fact that a partially polarized wave with stokes vector (I,Q,U,V)

can be decomposed into a completely polarized wave with stokes vector $[I_c', Q, U, V]^T$ and a unpolarized wave with stokes vector $[I_u, 0, 0, 0]^T$ where $I = I_c + I_u$ and $I_c^2 = Q^2 + U^2 + V^2$ uniquely.

The stokes reflection matrix R can be decomposed into M and N matrices as -

$$\begin{bmatrix} Ao+Bo & F & C & H \\ F & -Ao+Bo & G & D \\ C & G & Ao+B & -E \\ H & D & -E & Ao-B \end{bmatrix} = \begin{bmatrix} Ao+Bo^T & F^T & C & H \\ F^T & -Ao+Bo^T & G & D \\ C & G & Ao+B^T & -E^T \\ H & D & -E^T & Ao-B^T \end{bmatrix} + \begin{bmatrix} Bo^N & F^N & 0 & 0 \\ F^N & Bo^N & 0 & 0 \\ 0 & 0 & B^N & -E^N \\ 0 & 0 & -E^N & -B^N \end{bmatrix} \quad ..(2.3.4)$$

<u>where</u>

$$\begin{aligned} 2Ao(Bo^T + B^T) &= C^2 + D^2 \\ 2Ao(Bo^T - B^T) &= G^2 + H^2 \\ 2AoE^T &= CH-DG \\ 2AoF^T &= CG+DH \end{aligned} \qquad \underline{\text{and}} \qquad \begin{aligned} Bo &= Bo^T + Bo^N \\ B &= B^T + B^N \\ E &= E^T + E^N \\ F &= F^T + F^N \end{aligned} \quad ..(2.3.5)$$

The stokes reflection matrix M of a single target is irreducible in the sense that M cannot be further decomposed as sum of stokes reflection matrices corresponding to a collection ofsingle targets. This therom points to a fundamental limitation of traditional attempts at "sectionalizing" an object of complex shape into independent simpler shapes. Although at higher frequencies

when size to wave length ratio is large, such methods of computing RCS have had some success, the therom shows the futility of such attempts at lower frequencies.

The target scattering matrix, introduced in the previous section and stokes reflection **matrix** describe the same target scattering characterstics. Hence one should able to get one matrix given the other one. The conversion follows from following steps.

Voltage received at Rx terminals $V = \underline{b}'S\,\underline{a}$
where $\underline{a}$, $\underline{b}$ are Tx and Rx antenna polarizations, which converts voltage into electric field and vice-versa.

The received power is given by - $P_s = VV^* =$

$$(\underline{B}'\, S\underline{a})\quad(b'\, S\underline{a})^* \quad \ldots (2.3.6)$$

Simplification of above equation finally yields [ ]

$$Ps = h(b)'\ \mathbf{M}\ g(\underline{a})$$

where g ($\underline{a}$), h($\underline{b}$) stokes vectors of Tx and Rx antenna polarizations,
and M is the stokes reflection matrix defined above ..(2.3.7)

The elements of stokes reflection matrix M is related to TSM elements as follows

Let $$S = \begin{bmatrix} x_1+jx_2 & z_1+jz_2 \\ z_1+jz_2 & y_1+jy_2 \end{bmatrix} \quad \text{and} \quad M=\begin{bmatrix} m11 & m12 & m13 & m14 \\ m21 & m22 & m23 & m24 \\ m31 & m32 & m33 & m34 \\ m41 & m42 & m43 & m44 \end{bmatrix} \qquad \ldots(2.3.8)$$

$$m11 = \frac{1}{2}\left[x_1^2+x_2^2+y_1^2+y_2^2+2z_1^2+2z_2^2\right]$$

$$m22 = z_1^2 + z_2^2 - x_1y_1 - x_2 y_2$$

$$m33 = \frac{1}{2}\left[x_1^2+x_2^2+y_1^2+y_2^2-2z_1^2-2z_2^2\right]$$

$$m44 = z_1^2+z_2^2+ x_1y_1 + x_2y_2 \qquad \ldots(2.3.9)$$

$$m13 = m31 = \left[\frac{1}{2} \quad x_1^2 + x_2^2 - y_1^2- y_2^2\right]$$

$$m12 = m21 = z_1(y_2- x_2) - z_2(y_1 - x_1)$$

$$m14 = m41 = z_1(x_1- y_2) + z_2(x_2 + y_2)$$

$$m23 = m32 = z_2(x_1+ y_1)\ p - z_1(x_2+y_2)$$

$$m24 = m42 = (x_2y_1 - x_1y_2)$$

$$m34 = m43 = z_2(y_2 - x_2) + z_1(y_1 - x_1)$$

Given the Stokes reflection matrix M, one can obtain the TSM uniquely except for absolute phase using the above relations.

Later when we compute scattering matrix for random rough surfaces, the elements of TSM are no longer deterministic but are statistically described. In these cases, the computation of stokes reflection matrix can be done with slight modifications of above relations. Assuming that the scattering matrix elements are gaussian and neglecting the correlation bet' scattering matrix elements, and bet' real and imaginary parts of each element, we can arrive at following relations for the elements of stokes reflection matrix R.

$$m11 = \frac{1}{2}[(\bar{x}_1)^2+(\bar{x}_2)^2+(\bar{y}_1)^2+(\bar{y}_2)^2+2(\bar{z}_1)^2+2(\bar{z}_2)^2+Var(x_1)$$
$$+Var(x_2) + Var(y_1) + Var(y_2) + 2Var(z_1)+2Var(z_2)]$$

$$m22 = (\bar{z}_1)^2+(\bar{z}_2)^2- \bar{x}_1\bar{y}_1- \bar{x}_2\bar{y}_2+ Var(z_1) + Var(z_2)$$

$$m33 = \frac{1}{2}[(\bar{x}_1)^2+(\bar{x}_2)^2+(\bar{y}_1)^2+(\bar{y}_2)^2- 2(\bar{z}_1)^2 - 2(\bar{z}_2)^2+Var(x_1)$$
$$+Var(x_2)+Var(y_1)+Var(y_2)-Var(z_1)-2Var(z_2)]$$

$$m44 = (\bar{z}_1)^2 + (z_2)^2 + \bar{x}_1\bar{y}_1+\bar{x}_2\bar{y}_2+Var(z_1)+Var(z_2)$$

$$m13=m31= \frac{1}{2}[(\bar{x}_1)^2+(\bar{x}_2)^2-(\bar{y}_1)^2- (\bar{y}_2)^2 + Var(x_1)$$
$$+ Var(x_2) - Var(y_1) - Var(y_2)] \qquad ...(2.3.10)$$

$$m12=m21= \bar{z}_1(\bar{y}_2-\bar{x}_2) - \bar{z}_2(\bar{y}_1-\bar{x}_1)$$

$$m14 = m41 = \bar{z}_1 (\bar{x}_1+\bar{y}_1) + \bar{z}_2(\bar{x}_2+\bar{y}_2)$$

$$m23 = m32 = \bar{z}_2 (\bar{x}_1+\bar{y}_1) - \bar{z}_1(\bar{x}_2+\bar{y}_2)$$

$$m24 = m42 = \bar{x}_2\bar{y}_1 - \bar{y}_2\bar{x}_1$$

$$m34 = m43 = \bar{z}_2 (\bar{y}_2-\bar{x}_2) + \bar{z}_1(\bar{y}_1-\bar{x}_1)$$

where bar denotes mean values.

CHAPTER 3

SCATTERING FROM RANDOM ROUGH SURFACES

In this chapter, we shall discuss the methods of computing the TSM and stokes reflection matrices associated with a target in the form of random rough surface, using the scattering theories. The expressions derived are extended to time varying surfaces.

3.1 Review of Scattering theories:

When an e.m. wave with stokes vector $[I^i, Q^i, U^i, V^i]^T$ is incident on a object, it induces electric and magnetic currents thro'out the whole volume of the body. These currents in turn produces a field away from the object. This field is called scattered field with associated stokes vector $[I^s, Q^s, U^s, V^s]^T$. In general, instead of taking into account currents in the whole volume of the object, one can consider only the surface currents and compute the scattered field E^s, H^s at some point. This assumption is perfectly valid in cases where conductivity is very large and penetration depth is small. The theories calculating scattered field using only surface fields are called surface scattering theories

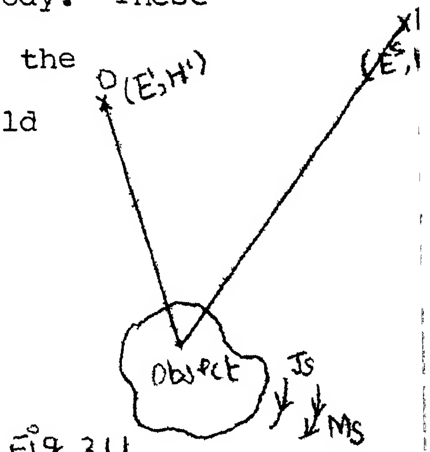


Fig 3.1

and these which calculate scattered field by considering fields in the whole volume of the object is called volume scattering theories.

The integral of Huygens-Kirchoff was modified to vector fields by Stratton-chu. The scattered field E^s at a point P away from the obstacle is expressed in terms of surface fields E and H as follows -

$$\vec{E}^s(P) = \iint [-j\mu\omega(\vec{n}\times\vec{H}) + (\vec{n}\times\vec{E})\times\nabla\psi + (\vec{n}.\vec{E})\nabla\psi] ds \quad .(3.1.1]$$

where $\psi = \frac{\exp(-jkr)}{r}$ and $\vec{n}$ is the unit vector normal to the surface and E, H are fields existing on the surface of the scattering object and can be computed using appropriate boundary conditions. The above equation is extended to Fraunhofer region by Silver (1948). The expression for E^s turned out to be the following -

$$\vec{E}^s = B\vec{K}_2 \times \iint [\vec{n}\times E - \eta\, \vec{K}_1\times(\vec{n}\times\vec{H})] \exp(j\, \vec{v}\cdot\vec{r})\, ds \quad ...(3.1.2]$$

where $\vec{K}_2 = \vec{k}_2/k_2$, $\vec{K}_1 = \vec{k}_1/k_1$ and

k_1, k_2 are wave numbers in medium (1) & (2)

and $B = \frac{j\, k \exp(jkR)}{4\, R}$

The above equation, when solved exactly gives

accurate results. But nevertheless, one has to make some approximations in order to evaluate the integral.

One such approximation, known as Kirchoff's approximation, essentially consists of approximating the boundary conditions. The approximation is as follows :-

"If the radius of curvature at a point is large compared to wave length of the incident radiation, then the fields at that point can be approximated as the fields that would exist, if that point is a part of infinite tangent plane."

Using this approximation one can compute $\vec{E}$ and $\vec{H}$ fields existing over the surface and hence $\vec{E}^s$ can be computed. We shall use the above approximation to compute scattered field $\vec{E}^s$.

As the angle of incident increases, one has to take into account the shadowing effects. For our purposes, we shall use the shadow-function proposed by Smith[26] for Gaussian surfaces.

There is another method called perturbation method to compute the scattered field $\vec{E}^s$. This method is applicable only to surfaces with small-scale roughness. This method gives more accurate results than previous method

for slightly rough surfaces. This method is widely used to model the sea at HF/MF/LF.

For some kind of surfaces, like sea at microwave frequencies, one has to use combination of above two methods. In this kind of surfaces, the small scale roughness is imposed upon large scale roughness. Hence one can compute the total scattered fields by computing the scattered field from small-scale rough surfaces and then averaging it over large-scale roughness.

In our derivations, we will start with the scattered field expression derived by Bahar (1981)

$$\vec{E}^{s} = G_0 j \int_A C^{s} T^{f} F T^{i}\, U(\vec{r}_s) \exp\left\{ j(\vec{n}^{s} - \vec{n}^{i}) \cdot \vec{r}_s \right\} \underline{\vec{E}}^{i}\, d\vec{A} \cdot \vec{n} \qquad ..(3.1.3)$$

where $\underline{E}^{s} = \begin{bmatrix} E^{SH} \\ E^{SV} \end{bmatrix}$; $\underline{E}^{i} = \begin{bmatrix} E^{iH} \\ E^{iV} \end{bmatrix}$;

$F = \begin{bmatrix} F^{HH} & 0 \\ 0 & F^{VV} \end{bmatrix}$ where F^{HH}, F^{VV} are modified reflection coefficients.

given by:-

$$F^{HH} = \frac{\epsilon - 1}{(\cos\theta + \cos_{\theta} \; \eta_r)^2}$$

$$F^{VV} = \frac{\cos^2\theta + \sin^2 \;)(1 - {}^{1}/_{\epsilon})}{(\cos\theta + \cos\theta\, \eta_r \;)^2} \qquad ...(3.1.4)$$

where θ is the angle of incidence

is the complex permittivity of the surface

given by $\epsilon_r + j60\,\sigma$

$\cos\theta_i = 1-\sin^2 {}_i$, $\eta_r = \dfrac{\sin\theta}{\sin\theta_i}$, $\eta_r = \sqrt{\dfrac{1}{\epsilon_r}}$

$U(rs)$ = shadow function = 1 for illuminated area

= 0 for non-illuminated area.

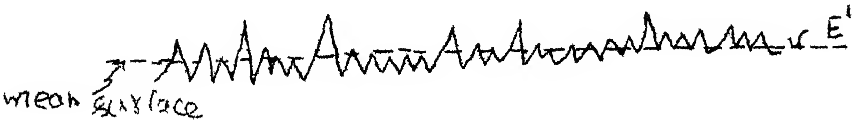
T^i, T^f are rotation matrices which rotates the incident field of reference-plane into local plane and vice-versa. They are given by -

$$T^i = \begin{bmatrix} \cos\phi_i & -\sin\phi_i \\ \sin\phi_i & \cos\phi_i \end{bmatrix} ; \quad T^f = \begin{bmatrix} \cos\phi & -\sin\phi_s \\ \sin\phi_s & \cos\phi_s \end{bmatrix} \qquad ..(3.1.5)$$

where ϕ_i is the angle bet' local plane and incident plane and ϕ_s is the angle bet' local plane and scatter plane.

Assuming a mean surface, in which $\underline{E}^i$ is constant, one can make $\underline{E}^i$ independent of area illuminated. Thus we can take $\underline{E}^i$ out of the integral in equation (3.1.3') and reqrite it as follows :-

$$E^s = G_0 \iint_A C_0 T^f F T^i \, U(\vec{r}_s) \exp\left\{ j(\vec{n}^s - \vec{n}^i) \cdot \vec{r}_s \right\} d\vec{A}\cdot\vec{n}\, E^i \qquad ..(3.1.6)$$



comparing with $\underline{E}^s = S\underline{E}^i$, we can write,

$$S = G_0 C_0 T^f F T^i \; U(\vec{r}_s) \exp\left\{ j(\vec{n}_s - \vec{n}^i) \cdot \vec{r}_s \right\} \; d\vec{A}\cdot\vec{n} \qquad ..(3.1.7)$$

We will use equation (3.1.5) as the starting equation in our derivations in next section.

3.2 Derviation of mean and variance of TSM for random rough surfaces.

Let us consider a rough surface with roughness height varying in Z-direction and is denoted by Z=h(x,y). If h(x,y) is deterministic, the solutions are straightforward and simple. But deterministic height variations are rarely realistic, model for practical applications. It is true that an individual area of terrain or seasurface at a given instant of time, is given by a deterministic, but not a random function. But because of difficulty in determining this function exactly and need to repeat the calculation for many other surfaces of same type, it is more convenient to regard the considered surface as a particular realization of a random function with given statistical properties.

In following derivations, we therefore restrict to the discussion entirely to random surfaces given by height distribution h(x,y). We make following assumption about the surface -

(1) The surface is assumed to be homogenous and isotropic. Hence the correlation function $c(x_d, y_d) = \langle h(x,y)\, h(x+x_d, y+y_d)\rangle$ depends only upon displacement bet' the points where correlation is taken. As the surface is isotropic, the correlation function will only depend upon the distance $l = (x_d^2 + y_d^2)^{1/2}$ where x_d, y_d are displacement bet' the points in x and y - directions, regardless of its direction.

(2) The co-ordinate system is defined in such a way as to make the x-y plane the mean of the surface, so that $\langle h(x,y)\rangle = 0$.

Let us start with the equation (3.1.3) given by -

$$S = G_o \iint C_o T^f F T^1 \exp\left\{jk(\vec{n}^s - \vec{n}^1)\cdot \vec{r}_s\right\} U(\vec{r}_s)\, d\vec{A}\cdot\vec{n} \qquad \ldots(3.2.1)$$

Let us evaluate mean and variance of S with following assumptions -

(1) The value of $\underline{D} = C_O T^f F T^1$ is constant through out the surface area illuminated. This assumption includes that the surface electrical properties are same thro'out the entire surface and angle ϕ is taken as average angle ϕ, which gives the average tilt of the entire surface.

(2) The function $U(r_s)$, though dependent upon xa and y co-ordinates, ist taken as average value.

Due to above two assumptions, $\underline{D}$ and $U(r_s)$ can be taken out of the integral and we are left with exponential term to be integrated -

$$S = Go\ \underline{D}\ \ U(\vec{r_s})\iint_A \exp\left\{jk\ (\vec{n}^s - \vec{n}^1)\ .\ \vec{r_s}\right\}\ dA\text{-}n$$

Let $\quad k\ (\vec{n}^s - \vec{n}^1) \quad = \quad = v_x\vec{a}_x + v_y\vec{a}_y + v_z\vec{a}_z$

$$= k(\sin\theta_s \cos\Delta_s \quad - \sin\theta \cos\Delta\ )$$

$$= k\ (\sin\theta_s \sin\Delta_s \quad - \sin\theta \sin\Delta\ )$$

...(3.2.2)

$$= k\ (\cos\theta_s + \ \cos\theta\ )$$

where (θ_s, Δ_s) is the scattered direction and

(θ, Δ) is the incident direction.

For back scatter,

$$\theta_s = \theta \quad , \quad \Delta_s = \pi \ , \quad \Delta = 0$$

$$\text{Hence, } v_x \quad = \quad - 2k \sin\theta$$

$$v_y \quad = \quad 0$$

...(3.2.3)

$$v_z \quad = \quad 2k\cos\theta$$

where θ is the incident angle.

The position vector $r_s = x\ a_x + y\ a_y + h(x,y)\ a_z$

Thus $k\,(\vec{n}^s - \vec{n}^1)\cdot\vec{r}_s = \vec{v}\cdot r_s = x v_x + y v_y + h(x,y)\,v_z$

Thus,

$$S = \frac{G_o\,\underline{D}\,U(r_s)}{A}\int_{-X}^{X}\int_{-Y}^{Y}\exp\left(jv_x x + jv_y y + jh(x,y)\,z\right)dxdy$$

taking expected value, we get

$$\langle s \rangle = \frac{G_o\,\underline{\underline{D}}}{A}\int_{-X}^{X}\int_{-Y}^{Y}\exp\left(jv_x x + jv_y y\right)\langle U(r_s)\,\exp(jh(x,y)v_z)\rangle$$

For gaussian surfaces,

$$\langle U(\vec{r}_s)\exp\, jh(x,y)v_z\rangle = \langle U(\vec{r}_s)\rangle\langle\exp(jh(x,y)v_z)\rangle$$

and identifying $\langle \exp(jh(x,y)v_z)\rangle = \chi(v_z)$ as characterstic function, we get,

$$\langle s \rangle = \frac{G_o\,\underline{\underline{D}}\langle U(r_s)\rangle}{A}\chi(v_z)\int_{-X}^{X}\int_{-Y}^{Y}\exp(jv_x x + jv_y)\,dxdy$$

$$\langle s \rangle = \frac{G_o\,\underline{\underline{D}}\langle U(r_s)\rangle}{A}\,\mathrm{sinc}\,(v_x X)\,\mathrm{sinc}\,(v_y Y)\,\chi(v_z)$$

Now let us calculate Var(s) s

$$\mathrm{Var}(s) = \langle SS^*\rangle - \langle S\rangle\langle S^*\rangle$$

$$\langle SS^*\rangle = \frac{G_o\,\underline{\underline{D}}\langle (U(r_s)\rangle^2}{A^2}\iint_{A_1}\iint_{A_2}\exp\left\{jv_x(x_1-x_2) + jv_y(y_1-y_2)\right\}\langle \exp\, jh_1(x,y) - jh_2(x,y)\rangle\,dx_1dy_1dx_2dy_2$$

$$\langle S\rangle\langle S^*\rangle = \frac{|G_o\,\underline{\underline{D}}\langle U(r_s)\rangle|^2}{A^2}\iint_{A_1}\iint_{A_2}\exp\left\{jv_x(x_1-x_2) + jv_y(y_1-y_2)\right\}\chi(v_z)\chi^*(v_z)\,dx_1dy_1dx_2dy_2.$$

$$\mathrm{var}(s) = \langle SS^*\rangle - \langle S\rangle\langle S^*\rangle$$

$$= \frac{|Go\, \underline{\underline{D}}\langle U(r_s)\rangle|^2}{A^2} \iint_{A_1}\iint_{A_2} \exp\left(jv_x x_d + jv_y y_d\right) \chi_2(v_z, -v_z) - \chi(v_z)\chi^*(v_z)\; dx_1 dy_1 dx_2 dy_2$$

where $x_d = x_1 - x_2$; $y_d = y_1 - y_2$, $(v_z, -v_z) = \langle \exp\, jh_1(x,y) - h_2(x,y)\rangle$

let $x_d = l\cos\beta$ and $y_d = l\sin\beta$

Substituting in above equation , we obtain after some simplifications

$$\mathrm{var}(s) = \frac{2\;|Go\, \underline{\underline{D}}\langle U(r_s)\rangle|^2}{A^2} \int_0^\infty J_o(v_{xy} l)[\chi_2(v_z, -v_z) - \chi(v_z)\chi^*(v_z)]\, l\, dl$$

For a gaussian surface, the height distribution is given by-

$$W_1(z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-z^2/2\sigma^2\right)$$

$$W_2(z) = \frac{1}{2\pi\sigma^2(1-c(l))^{\frac{1}{2}}} \exp - \frac{(z_1^2 - 2c(l)z_1 z_2 + z_2^2)}{2\sigma^2(1-c(l))}$$

where (l) is the correlation function.

Therefore

$$\chi(v_z) = \exp\left(-\tfrac{1}{2}\sigma^2 v_z^2\right)$$

$$\chi_2(v_z, -v_z) = \exp\left\{-\sigma^2 v_z^2 (1-c(l))\right\}$$

Substituting in equation (3.2.4), we get

Substituting for v_x, v_y and v_z from equation (3.2.3) we get,

$$\langle s \rangle = G_o \underline{D} \langle U(r_s) \rangle \ \mathrm{sinc} \ (-2k \sin\theta X) \exp(-2 \sigma^2 \kappa^2 \cos^2\theta) \quad ..(3.2.8)$$

substituting (v_z), $(v_z, -v_z)$ values in equation (3.2.6) we obtain,

$$\mathrm{var}(S) = \frac{2\pi |G_o \underline{\underline{D}} \langle U(r_s)\rangle|^2}{A^2} \int_0^\infty J_o(v_{xy} l) \left[\exp -\sigma^2 v_z^2 (1-c(l)) - \exp(-\sigma^2 v_z^2) \right] l\,dl \quad ..(3.2.9)$$

Let us assume a gaussian correlation function.

Thus $c(l) = e^{-l^2/T^2}$

where T is the correlation distance.

Let $g = v_z^2 \sigma^2$

Thus $\exp\left[\sigma^{-2} v_z^2 (1-c(l))\right] = \exp\left[-g(1-c(l))\right]$

expanding $\exp\left[-gc(l)\right] = \sum_{m=0}^{\infty} \frac{g^m e^{-ml^2/T^2}}{m}$

where we have substituted $c(l) = e^{-l^2/T^2}$

Thus the expression for variance becomes,

$$\mathrm{var}(s) = \frac{2\pi |G_o \underline{D} \langle U(r_s)\rangle|^2}{A^2} \int_0^\infty J_o(v_{xy} l) \, e^{-g} \sum_{m=1}^{\infty} \frac{g^m e^{-m^2/T^2}}{m!} \, l\,dl \quad ..(3.2.10)$$

interchanging summation and integrations, we get,

$$\mathrm{var}(s) = \frac{\pi|G_o\, D\, \langle U(r_s)\rangle|^2}{A^2}\, e^{-g} \sum_{m=1}^{\infty} \frac{g^m}{m\,m!} \exp\left\{-v_{xy}^2 T^2/4m\right\} \quad \ldots (3.2.11)$$

The above equation is valid for $g \ll 1$ and $g \gtrsim 1$.

For the surfaces with $g \gg 1$, instead of expanding $\exp\left\{- gc(1)\right\}$, we expand the correlation function

Thus $c(1) = e^{-1^2/T^2} \approx (1+1^2/T^2)$

substituting in equation (3.2. 9) and simplification yields.

$$\mathrm{var}(s) = \frac{2\pi|G_o\, D\, \langle U(r_s)\rangle|^2}{A^2 g} \exp -\left\{\frac{v_{xy}^2\ T^2}{4v_z^2\ \sigma^2}\right\} \quad (3.2.12)$$

3.3 Shadow function.

As we saw earlier, shadowing effects become very important, as we approach the grazing angles. In order to compute this function, we use the expression derived by Smith , for gaussian surfaces. In expressions derived in previous sections, we have taken shadowing into account thro' the function $\langle U(\vec{r}_s)\rangle$

Basically, $\langle J(\vec{r}_s)\rangle$ is the probablity that a point not being shadowed. The shadowing includes the invisiblity of the point to the transmitter, receiver or both. The expression given by Smith is as follows -

$$U(F,\theta) = \frac{1 - 1/2\ \mathrm{erfc}\left(\frac{\mu}{\sqrt{2\omega}}\right)}{\Lambda(\mu) + 1}$$

where $\mu = \cot\theta$ $\quad w^2 = |C''(o)|$ and

$$2\Lambda(\mu) = \left(\frac{2}{\pi}\right)^{1/2} \frac{w}{\mu} e^{-\mu^2/2w^2} - \operatorname{erfc}(\mu/\sqrt{2}w)$$

and θ is the angle of incidence and $c''(0)$ is the second derivative of the correlation function evaluated at origin.

The above expression is valid for any correlation function . Let us evaluate the above function for a gaussian correlation function.

$$c(l) = e^{-l^2/T^2}$$

$$|c''(o)| = 2/T^2$$

and $\quad w = \sqrt{2}/T$

Substituting in above expression, we get,

$$U(F,\theta) = \frac{1 - \frac{1}{2}\operatorname{erfc}\ (T\cos\theta)/2}{\Lambda(\cot\theta) + 1} \qquad \text{....(3.3.2)}$$

we will use this expressions in our equations derived above for $\langle U(\vec{r}_s)\rangle$.

$$\langle U(\vec{r}_s)\rangle = U(F,\theta).$$

Thus to evaluate the mean and variance of TSM, we should know following things.

1) physical properties of the surface (permittivity, permeability, conductivity);

2) angle of incidence
3) correlation function c(l) and correlation distance T
4) surface roughness height deviation
5) Area illuminated.

3.4 Effect of temporal variation of the rough surface

In the previous sections, we discussed scattering from a time invariant surface. In this section, we extend the results to include the time variation of the surface. In this case, the time variation of the surface is slow compared to the velocity of electromagnetic radiation. Hence we can apply quasi-static approximation to this case. By this approximation, all the equations derived in sec.3.2 are still applicable with an additional term for doppler spectrum.

Let the height deviation in z-direction is given by $z = h(x,y,t)$. In this case the equation for S(t) is modified as follows :

$$S(t) = \frac{G_o \underline{\underline{D}}}{A} U(r_s) \int_{x}^{x} \int_{-y}^{y} \exp(jv_x x + jv_y y) \exp jh(x,y,t)v_z \, dxdy \quad ..(3.4.1)$$

and expected value of S(t) becomes,

$$\langle S(t) \rangle = \frac{G_o \underline{\underline{D}}}{A} \chi(v_z,t) \operatorname{sinc}(v_x x) \operatorname{sinc}(v_y y) \quad ...(3.4.2)$$

where $\chi(v_z,t) = \langle \exp jh(x,y,t) v_z \rangle$ is the characterstic function, which is dependent on time.

The variance expression (3.2.6) gets modified time-varying surface as follows

$$\mathrm{var}\{S(t)\} = \frac{|Go\underline{\underline{D}}\langle U(\vec{r}_s)\rangle|^2}{A^2} \int_0^\infty Jo(v_{xy}l)[X(v_z, -v_z, c(l),\tau)) - X(v_z,t]\, X^*(v_z,t)]\, l\,dl \quad \ldots(3.4.3)$$

where $l^2 = x_d^2+y_d^2$ and $v_{xy}^2 = v_x^2+v_y^2$.

The height deviation $h(x,y,t)$ can be decomposed by using fourier integral as follows:

$$h(x,y,t) = \iint_{-\infty}^{\infty} dv_x dv_y\, h(v_x,v_y)\, e^{j[xv_x+yv_y-\Omega(v_x,v_y)t]} \quad \ldots(3.4.4)$$

The above representation indicates that the surface perturations can be represented by monochromatic plane waves travelling with various wave vectors and frequencies .

Hence we obtain following representation for space-time correlation function:

since $c_\omega(x_d,y_d,\tau) = \langle h(x+x_d, y+y_d, t+\tau)h(x,y,t)\rangle$

we get,

$$c(x_d,y_d,\tau) = \int\int_{-\infty}^{\infty} dv_x dv_y\, w(v_x,v_y)\, \exp[j\, v_x x_d + j v_y y_d - \Omega(v_x,v_y)\ ] \quad \ldots(3.4.5)$$

where $w(v_x,v_y)$ is a real, non-negetive function and is called rough surface spectrum.

The frequency Ω of each elementary harmonic is independent of the direction of its propagation for a homogenous and isotropic surface. It is related to the

spatial period $2\pi/v_{xy}$, = (v_{xy}). The explicit form of this dependence (the 'dispersion-relation') is defined by the specific form of the equation of motion.

Thus for a surface with dispersion relation (v_{xy}) the characterstic function becomes,

$$x(v_z,t) = \bar{e}^{1/2\ \sigma^2 v_z^2}\ \bar{e}^{1/2\ (v_{xy})t}$$

and $X_2(v_z,-v_z,c(1,\ )) = e^{-\sigma^2 v_z^2(1-c(1,\ ))}$

Substituting in above equation (4.5 2), we get,

$$\text{var } S(t) = \frac{|\underline{\underline{G}}o\underline{\underline{D}} \langle U(r_s)\rangle|^2}{A^2} \int_o^\infty Jo(v_{xy}l)[\exp -\sigma^2 v_z^2(\ -c(l,\ )) - \exp(\ \sigma^2 v_z^2)]\ ldl$$

Let us evaluate above equation for two cases.

<u>case</u> (i) $v_z\sigma \ll 1,\ v_z\sigma\ \ 1$

$$\text{var } S(t) = \frac{|\underline{\underline{G}}o\underline{\underline{D}}\langle U(r_s)\rangle|^2}{} \int_o^\infty Jo(v_{xy}l)\ \bar{e}^{\sigma^2 v_z^2} \left[\sum_{m=o}^{\infty} \frac{(v_z^2\ \sigma^2)^m}{m}\ c^m(l,\ )-1\right] ldl$$

$$= \left|\frac{\underline{\underline{G}}o\underline{\underline{D}}\langle U(r_s)\rangle|^2}{A^2} \int_o^\infty Jo(v_{xy}l)\ \bar{e}^{\sigma^2 v_z^2} \sum_{m=1}^{\infty} \frac{(v_z^2\sigma^2)^m}{m} c^m(l,\ )ldl.\right.$$

interchanging summation and integration signs, we get,

$$\text{var } s(t) = \frac{|\underline{\underline{G}}o\underline{\underline{D}}\langle U(r_s)\rangle|^2}{A^2}\ \bar{e}^{\sigma^2 v_z^2} \sum_{m=1}^{\infty} \frac{(v_z^2\sigma^2)^m}{m}$$

$$\int_o^\infty Jo(v_{xy}l)c^m(l,\ )ldl.$$

For m=1, this equation reduces to first-order perturvation solution which is valid for $v_z\sigma \ll 1$. For $v_z\sigma\ \ 1$, we have to

take sufficient number of terms and sum it to obtain $\mathrm{var}\{S(t)\}$.

case (ii) $v_z\sigma \gg 1$.

In this case instead of expanding the exponential, we expand the correlation function $c(1,\tau)$. For the purposes of illustration, we shall consider first a 2-dimensional surface with $z = h(x,t)$ and later extend the results to 3-dimensional case.

For two dimensional case, equation (3.4.3) is modified as follows : [ ]

$$\mathrm{var}\{S(t)\} = \frac{|G_0\underline{\underline{D}}\langle U(r_s)\rangle|^2}{A^2} \int_0^{\infty} e^{jx_d v_x} [\chi_2(v_z,-v_z,c(1,\tau)) - \chi(v_z,t)\,\chi^*(v_z,t)]\, dx_d \quad \ldots(3.4.6)$$

expanding the correlation function $c(x_d,\tau)$ near origin and taking only upto second order terms, we have,

$$c(x_d,\tau) = 1 + 1/2\, c_{11}x_d^2 + c_{12}x_d\tau + 1/2\, c_{22}\tau^2$$

$$\text{where } c_{11} = \frac{\partial^2 c(x_d,\tau)}{\partial x_d^2}\ ;\ c_{12} = \frac{\partial^2 c(x_d,\ )}{\partial x_d \partial\tau} \quad \ldots(3.4.7)$$

$$c_{22} = \frac{\partial^2 c(x_d,\ )}{\partial^2\tau}$$

As $v_z\sigma \gg 1$, we shall neglect $|\chi(v_z,t)|^2$ term.

Thus equation (3.4.) becomes,

$$\mathrm{var}\left\{s(t)\right\} = \frac{|G_oD\langle U(r_s)\rangle|^2}{\phi L^2} \int_0^\infty e^{jv_x x_d} \exp -\sigma^2 v_z^2 \,(1/2c_{11}x_d^2 + c_{12}x_d + 1/2\, c_{22}\tau^2)\, dx_d \quad ..(3.4.8)$$

performing integration, by completing squares in the exponent and using the standard integral -

$$\int_0^\infty \bar{e}^{dx^2}\, dx = \sqrt{\frac{\pi}{d}}\text{ , we finally obtain,}$$

$$\mathrm{var}\left\{s(t)\right\} = \frac{|G_oD\langle U(r_s)\rangle|^2}{A^2} \sqrt{\frac{\pi}{2}} \frac{1}{\sigma v_z (c_{11})^{1/2}} \exp\left\{\frac{v_x^2}{2v_z^2\sigma^2 c_{11}}\right\}$$

$$\exp \frac{\sigma^2 v_z^2}{2\, c_{11}} (c_{11}\, c_{22} - c_{12}^2)^{\,2} \quad ...(3.4.9)$$

since autocorrelation $\langle s(t+\ ) s^*(t)\rangle$ - $\mathrm{var}\{s(t)\}$ for this case, the Fourier transform of above equation is given the spectral density. Thus,

$$s(\omega)\ \alpha\ \frac{1}{\sqrt{\pi}\,\Delta\omega} \exp\left\{-\frac{(\omega - \omega_m)}{(\Delta\omega)^2}\right\}$$

$$\text{where } (\Delta\omega_m)^2 = \frac{\sigma^2 v_z^2}{2c_{11}} (c_{11}c_{22} = c_{12}^2)$$

$$\omega_m = v_x \frac{c_{12}}{c_{11}} \quad ...(3.4.10)$$

The above equation is valid for in general, any dispersion relation Ω . The scattered held spectrum is

a gaussian spectrum with mean and variance values depending upon the surface roughness frequency, aspectangle and the kind of dispersion relation assumed.

For three dimensional case, (i.e.) $z = h(x,y,t)$ the above equation modify to .- [ ]

$$S(\omega) = \frac{\pi |G_o \underline{\underline{D}} \langle U(r_s)\rangle|^2}{A\, v_z^2 \sigma^2 |c_{11}|} \exp\left\{ \frac{v_{xy}^2}{2v_z^2 \sigma^2 |c_{11}|} \right\} \cdot \frac{1}{\sqrt{\pi \Delta\omega}} \exp\left\{ -\frac{(\omega-\omega_m)^2}{(\Delta\omega)^2} \right\}$$

where $\omega_m = \frac{D_1}{d}$ and $(\Delta\omega)^2 = \frac{(v_z \sigma)^2}{2} \frac{D}{d}$ ...(3.4.11)

and

$$D = \begin{vmatrix} c_{xx} & c_{xy} & c_{x\tau} \\ c_{yx} & c_{yv} & c_{y\tau} \\ c_{\ x} & c_{\ y} & c_{z\tau} \end{vmatrix} \; ; \quad d = \begin{vmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{vmatrix}$$

$$D_1 = \begin{vmatrix} v_x & v_y & o \\ c_{xx} & c_{xy} & c_{x\tau} \\ c_{yx} & c_{yy} & c_{y\tau} \end{vmatrix}$$

where the subscripts for C in indicates the corresponding second derivatives of $c(xd, yd, \tau)$ where $x_d = y_d = \tau = o$.

CHAPTER 4

APPLICATION TO PRACTICAL SURFACES

In this chapter TSM and Stokes reflection matrices are numerically computed using the results obtained in sections 3.2 and 3.3, for the cases of an asphalt road and sea-surface. The Stokes reflection matrix obtained is decomposed using the decomposition theorem discussed in sec.2.3. The TSM corresponding to completely polarized component is calculated and the associated target parameters are obtained. The variation of these parameters with frequency, aspect angle and surface roughness are studied. For sea-surface, the spectrum of the scattered field is also obtained.

4.1 Application to asphalt road:

The asphalt road has a complex permittivity of $4.3 + j0.15$ and x-band and $2.5 + j0.65$ at K_a-band[23]. The height deviation $k\sigma$ at these frequencies is assumed to be 0.5 and the correlation distance kT is assumed to be 0.5. Using these numerical values TSM and Stokes reflection matrices are computed as indicated above. The eigenvectors and eigenvalues of the problem $S\underline{x} = t\underline{x}^*$ are determined from the solution of the associated eigenvalue problem $S^+S\underline{x}= \underline{x}$. This method of computing the

eigenvalues and eigenvectors is givenin.appendix A. From these, the associated target parameters are then computed and their variation with surface roughness and aspectangle are plotted in the graphs [4.1.1] and [4.1.2]. As the linear polarization gives the maximum return, the values of τ and ψ are constant and hence they all not plotted.

4.2 Application to sea-surface

Sea surface, due to external forces like wind, is a time varying surface. Due to this, there is dispersion in frequency of scattered hold.

The surface parameters of the sea are obtained from various sources as indicated below:

The permittivity of sea-water is obtained using Debye's formula given by -

$$= \epsilon_{\infty} + \frac{\epsilon_s - \epsilon_{\infty}}{1+\omega^2\tau^2} \qquad \ldots(4.2.1)$$

$$= \frac{\omega\tau(\epsilon_s - \epsilon_{\infty})}{1+\omega^2\tau^2}$$

and $\epsilon_r = \epsilon' - j\epsilon''$

where ϵ_s is the static dielectric constant.

ϵ_{∞} is the dielectric constant at infinite frequency

τ is the relaxation time

$\omega = 2\pi f$ and f is the frequency at which dielectric constant is to be obtained

σ is the conductivity

ϵ_0 is the free space permittivity.

At 30°c, $\tau = 7.2 \times 10^{-2}$ sec; $\epsilon_s = 68.0$; $\epsilon_\infty = 4.9$;
$\sigma = 1.567$ mhos/m [20]

The roughness height is taken to be the significant wave height. Significant wave height can be related to wind velocity as follows [19]

$$H_s = 7.15 \times 10^{-3} \, W_s^{2.5} \qquad \ldots(4.2.2.)$$

where ω_s is the wind velocity in m/sec.

The correlation length T is estimated from the equation

$$T = (0.556)^2 \, W_s^2 \qquad \ldots(4.2.3)$$

Using above estimated values, the mean and variance of TSM and hence Stokes reflection matrix can be computed. The associated target parameters are obtained and their variations with aspect angle, frequency and wind velocity are obtained. It is found that in this case also, linear polarizations give rise to maximum returns. Hence and are constants. Also it is observed that $\gamma = 0^\circ$ for the sea-surface. Hence only the variations of m and γ are plotted in Figs. 4.2 and 4.2.2.

The temporal variations of the sea-surface give rise to the doppler spectrum of the scattered waves. As discussed earlier, the dispersion relation (v_x,v_y) of the surface, modifies the surface spectrum $W(v_x,v_y)$ of the sea as follows: [9]

$$W(v_x,v_y,\tau) = W(v_x,v_y)\ e^{-j\Omega(v_x,v_y)\tau} \qquad \ldots(4.2.4)$$

For back scatter, $v_x = 2k\sin\theta$ and $v_y = 0$.

We shall assume a gaussian spectrum for sea-surface.

$$W(v_x,0) = \frac{T}{\sqrt{\pi}}\ \exp\left(-v_x^2T^2\right) \qquad \ldots(4.2.5)$$

where T is the correlation length

At low wind velocities when the wind is uniformly blowing the sea-surface can be assumed to have a linear dispersion. This assumption is valid, also for high wind velocities, at low frequencies. However, when the frequency becomes higher, or when the wind velocity is random, the dispersion becomes non-linear. In order to study the spectrum for these two situations, let us consider two kinds of dispersion relations. since the surface is assumed to be homogenous and isotropic, dispersion relation depends only on $v_{xy} = \sqrt{v_x^2+v_y^2}$ and is equal to v_x for back scatter case.

<u>case (i)</u> <u>linear dispersion</u>

In this case, the dispersion relation is given by

$\Omega(v_x) = uv_x = -\ 2ku\sin\theta$ where u is the velocity of the

scatterers, k is the wave number and θ is the angle of incidence.

The velocity of scatterers is estimated from the relation given by : [24]

$$uU = \sqrt{gv_x} + 0.04\,W_s^{1.5} + 0.02W_s \qquad \ldots(4.2.6)$$

where W_s the wind velocity in m/sec. and g is the acceleration due to gravity. In the above equation, the first term corresponds to the effects due to gravity, second term represents effect due to orbital velocity and last term denotes wind effect.

The spectrum is computed using the equations (3.1.10). It is found that, for this case, the spectrum is a just a line shifted by the frequency $\omega_m = \mu v_x$. The variation of ω_m with aspect angle, incident frequency and velocity of the wind is plotted in graphs [4.2.--] to [4.2.--]

<u>case (ii)</u> <u>Non-linear dispersion</u>:

For non-linear dispersion,

$$\Omega(v_x) = uv_x + av_x^2$$

where $u = \left.\dfrac{d}{dv_x}\right|_{v_x=0}$ and $a = \left.\dfrac{d^2}{dv_x^2}\right|_{v_x=0}$

The value of u is taken as velocity of scatterers.

Since ω has the dimensions of radians/sec. and v_x has radians/m, a has a dimension of m^2/sec. In order to fit this dimension, the following relation for a is proposed:

$$a = D\mu T \qquad \ldots\ldots.(4.2.7)$$

where D is a constant μ is the velocity of scatters and T is the correlation length.

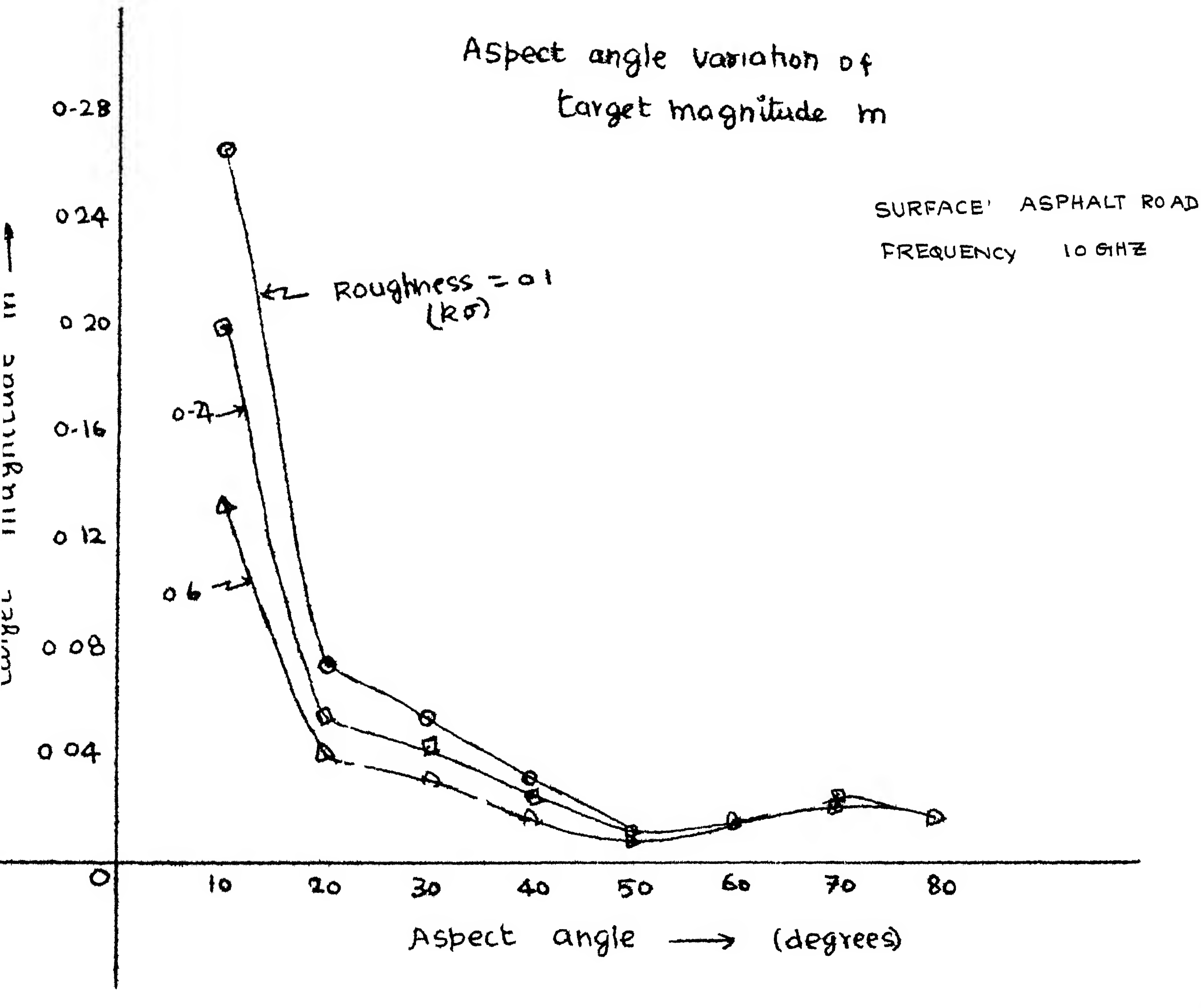
The velocity μ is computed using the relation(4.2.6). Using the above dispersion relation, the mean and variance of the scattered field frequency spectrum are obtained as follows :

$$\omega_m = \mu v_x$$

$$\text{and } (\Delta\omega_m)^2 = \frac{3}{8}\left(\frac{a^2}{T^4}\right)(v_z\,\sigma)^2 \qquad \ldots\ldots\ (4.2.8)$$

The standard deviation $\Delta\omega$ of the doppler is related to the half-power band width as [25]. $\Delta\omega_m = 0.42\,\Delta\omega$..(4.2.9) where $\Delta\omega$ is the half-power bandwidth in radians/sec.

The variations of the mean doppler ω_m and half-power bandwidth $\Delta\omega$ with aspect angle, with wind velocity as parameter are plotted in fig. [4.2.3] to [4.2.8]



Aspect angle variation of target magnitude m
SURFACE' ASPHALT ROAD
FREQUENCY 10 GHZ
Roughness = 0.1 (kσ)
0.7
0.6
0.28
0.24
0.20
0.16
0.12
0.08
0.04
0
10
20
30
40
50
60
70
80
Aspect angle ⟶ (degrees)

Fig 4.11

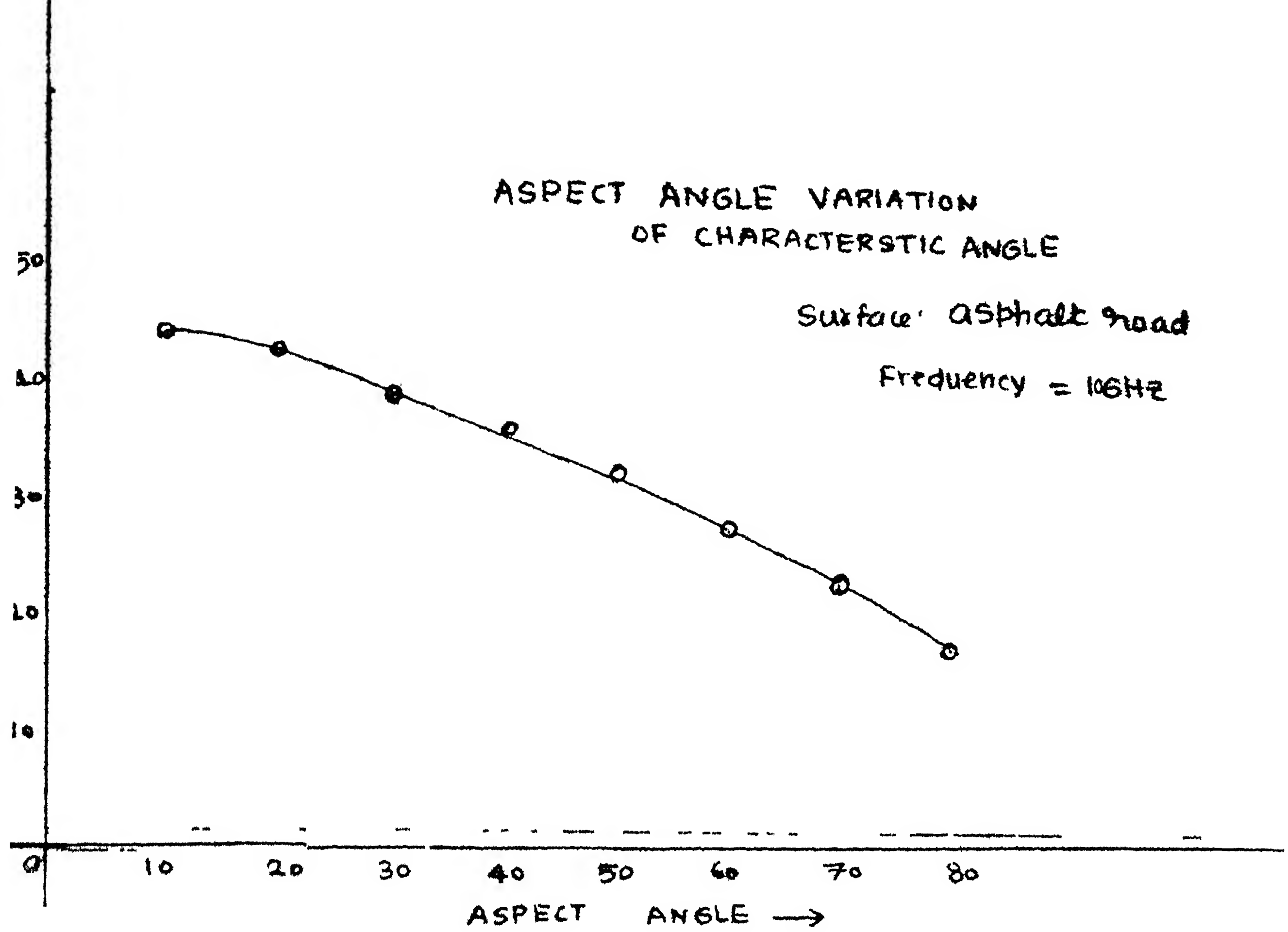
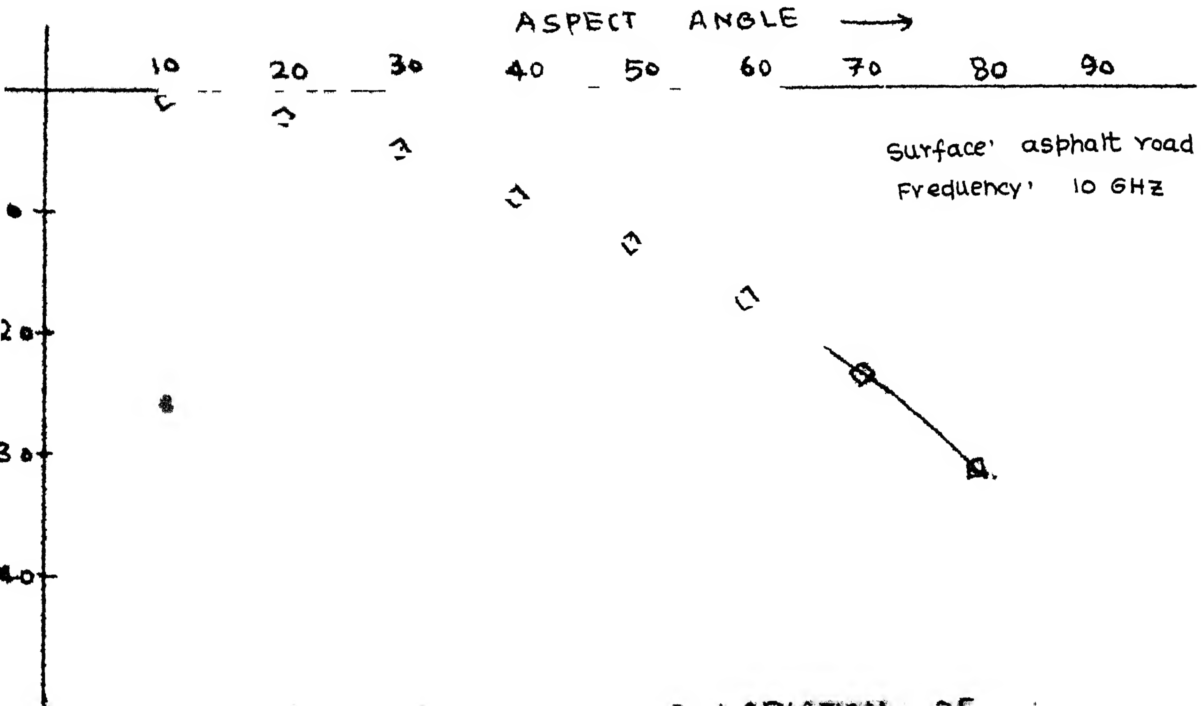

ASPECT ANGLE VARIATION
OF CHARACTERSTIC ANGLE
Surface: asphalt road
Frequency = 10GHz
50
40
30
20
10
0
10
20
30
40
50
60
70
80
ASPECT ANGLE →

Fig 4 2 2 a


ASPECT ANGLE →
10
20
30
40
50
60
70
80
90
Surface: asphalt road
Frequency: 10 GHz
20
30
40

ASPECT ANGLE VARIATION OF m

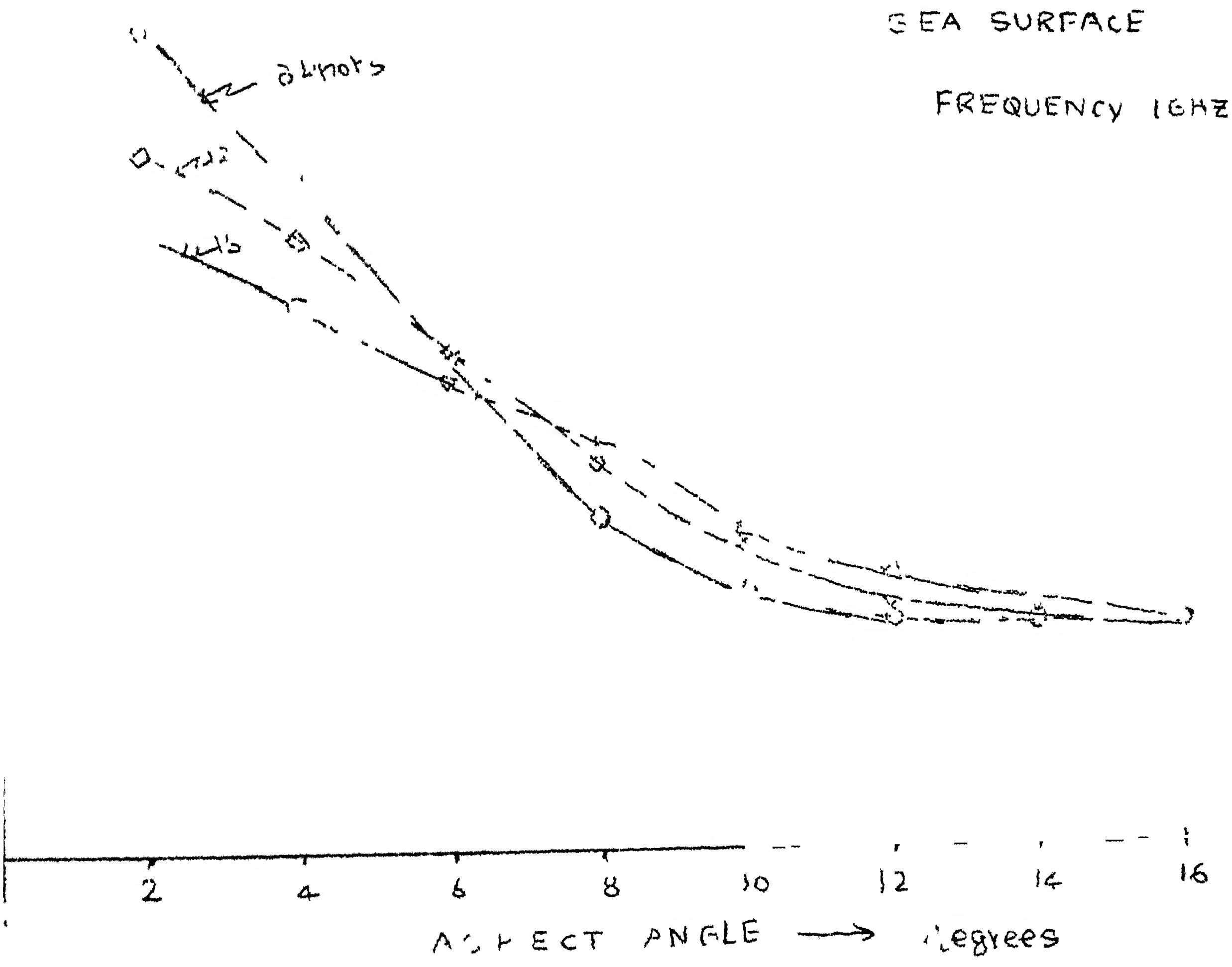


Fig 3 4 2 1

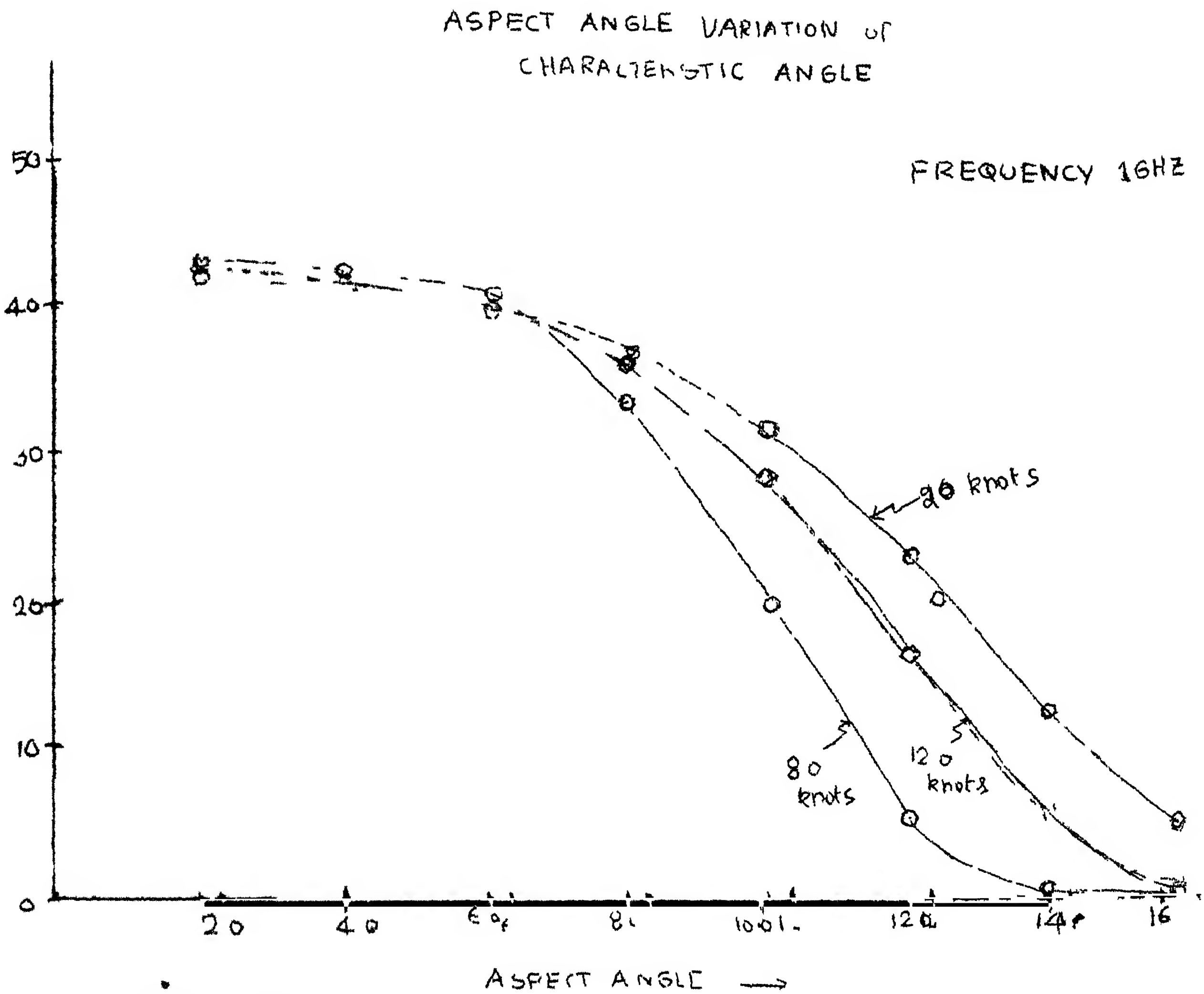

ASPECT ANGLE VARIATION OF
CHARACTERISTIC ANGLE
FREQUENCY 1GHZ
50
40
30
20
10
0
20
40
60
80
100
120
140
16
20 knots
80 knots
120 knots
ASPECT ANGLE

Fig 4 2 2

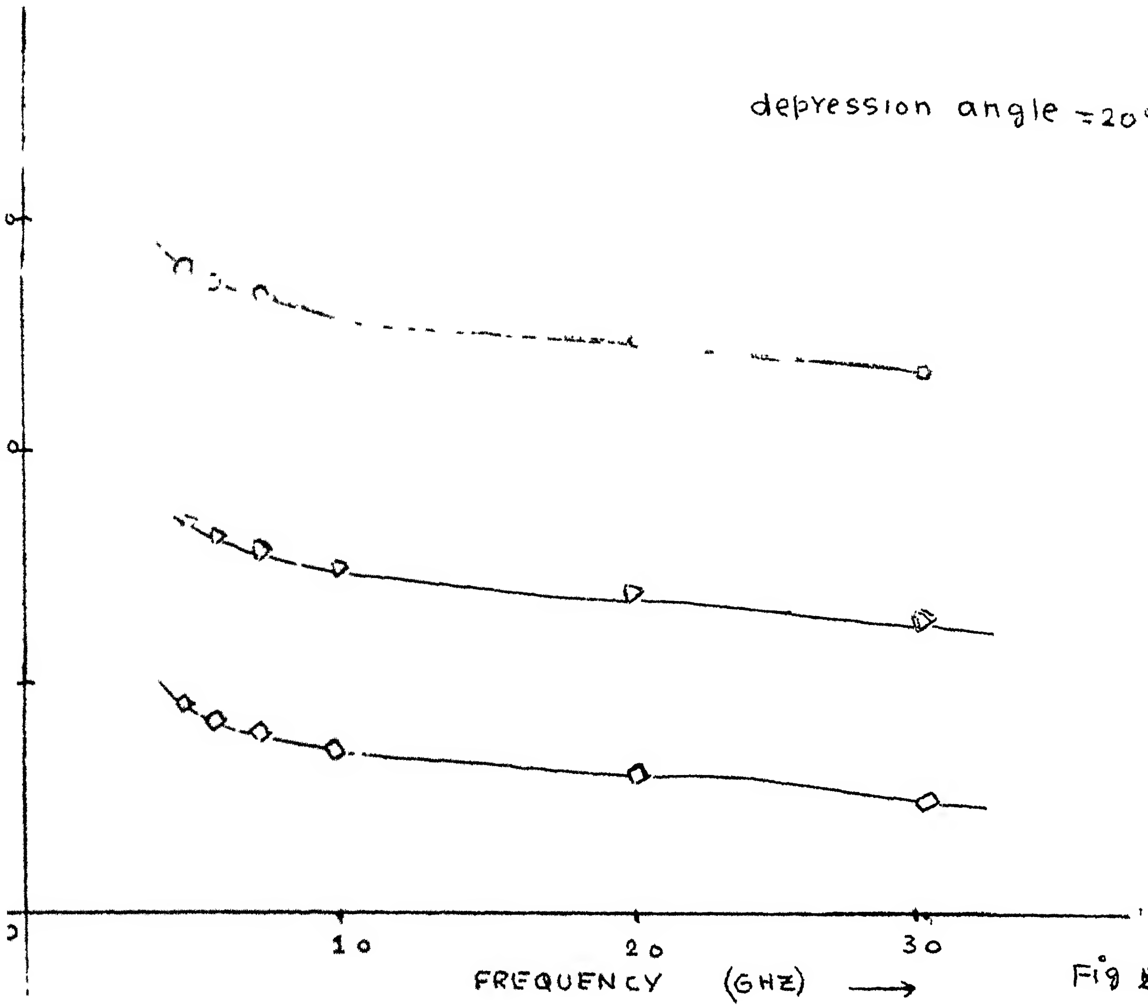
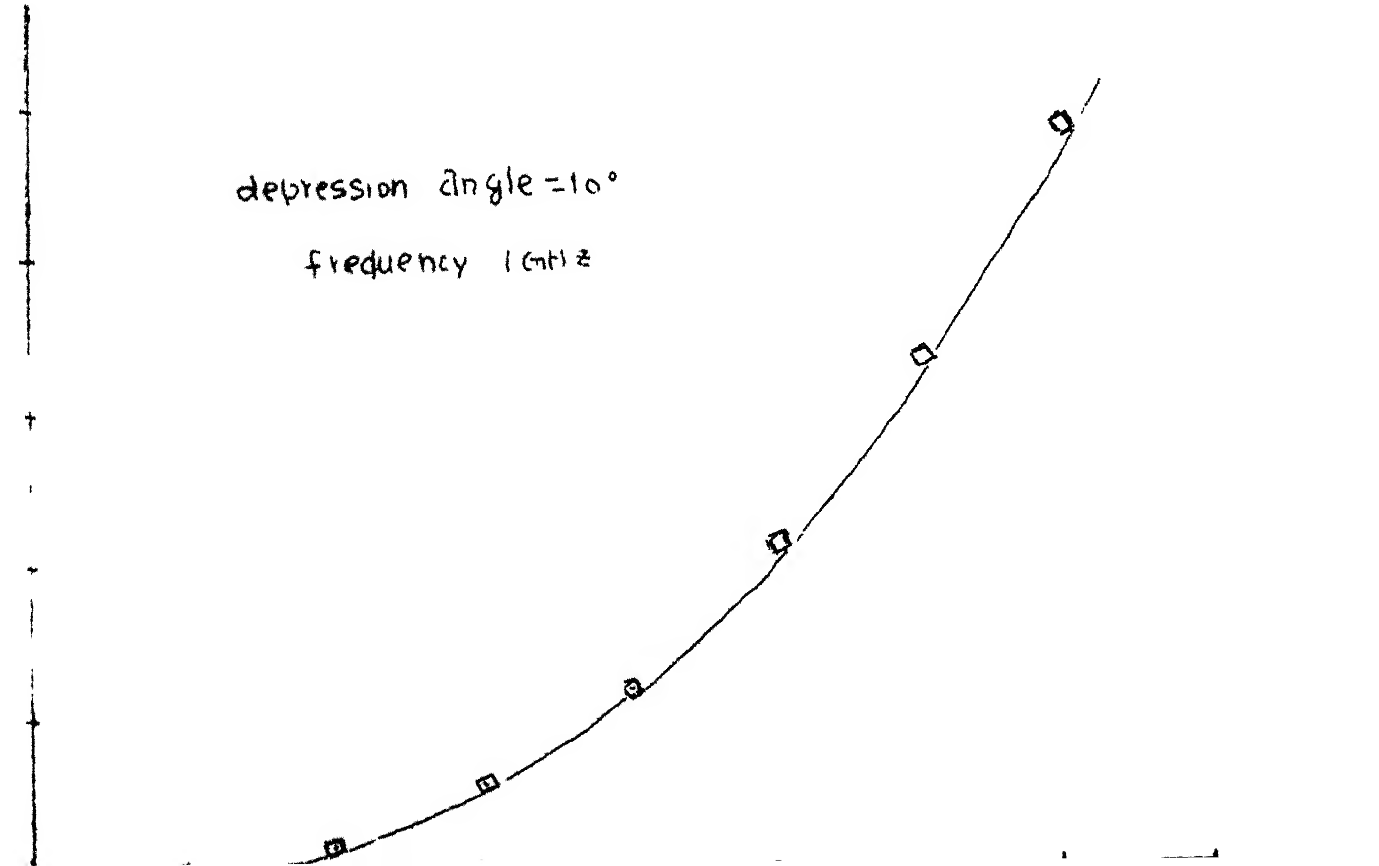

depression angle =20°
10
20
30
FREQUENCY (GHZ) →

Fig 4.23


depression angle =10°
frequency 1GHz

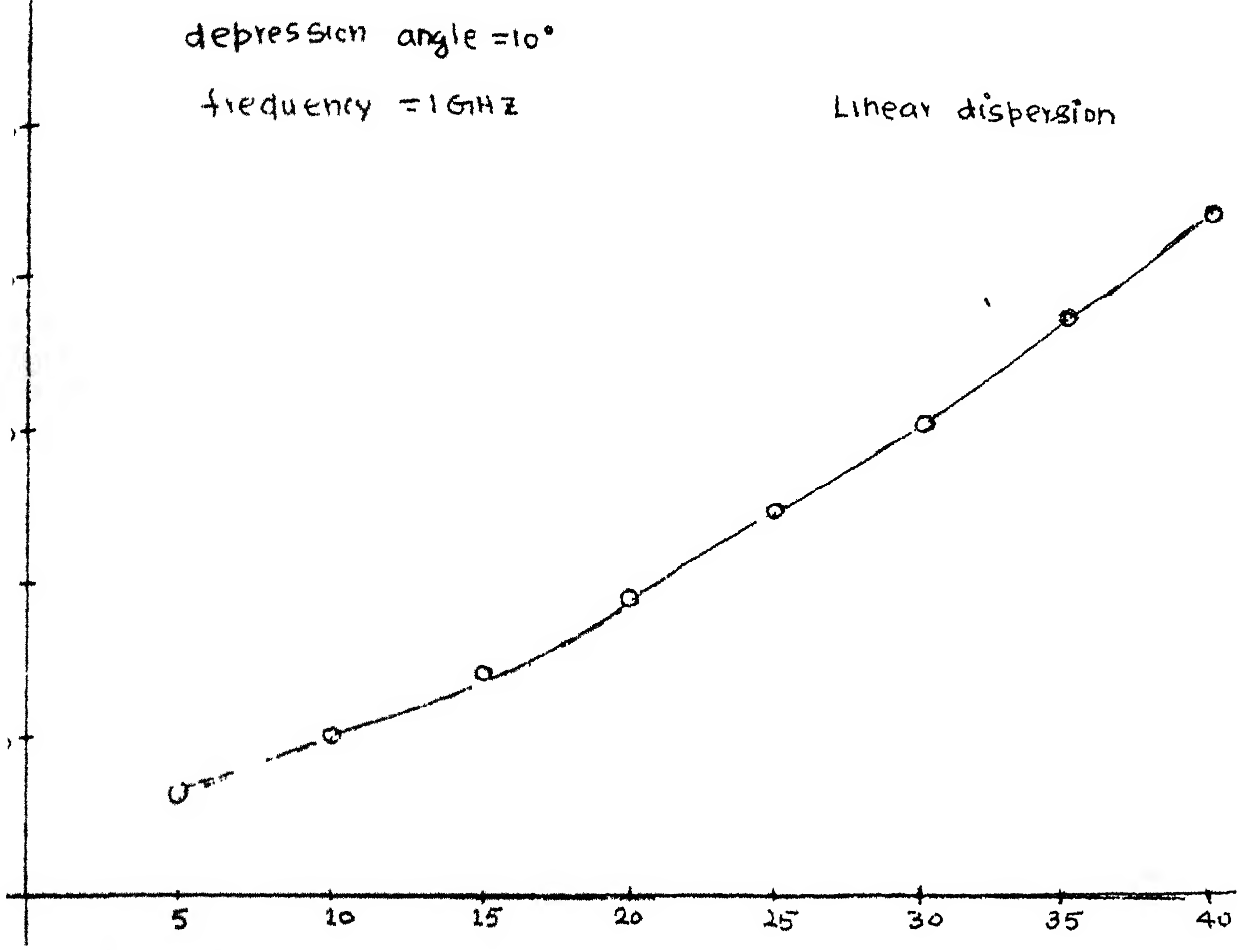
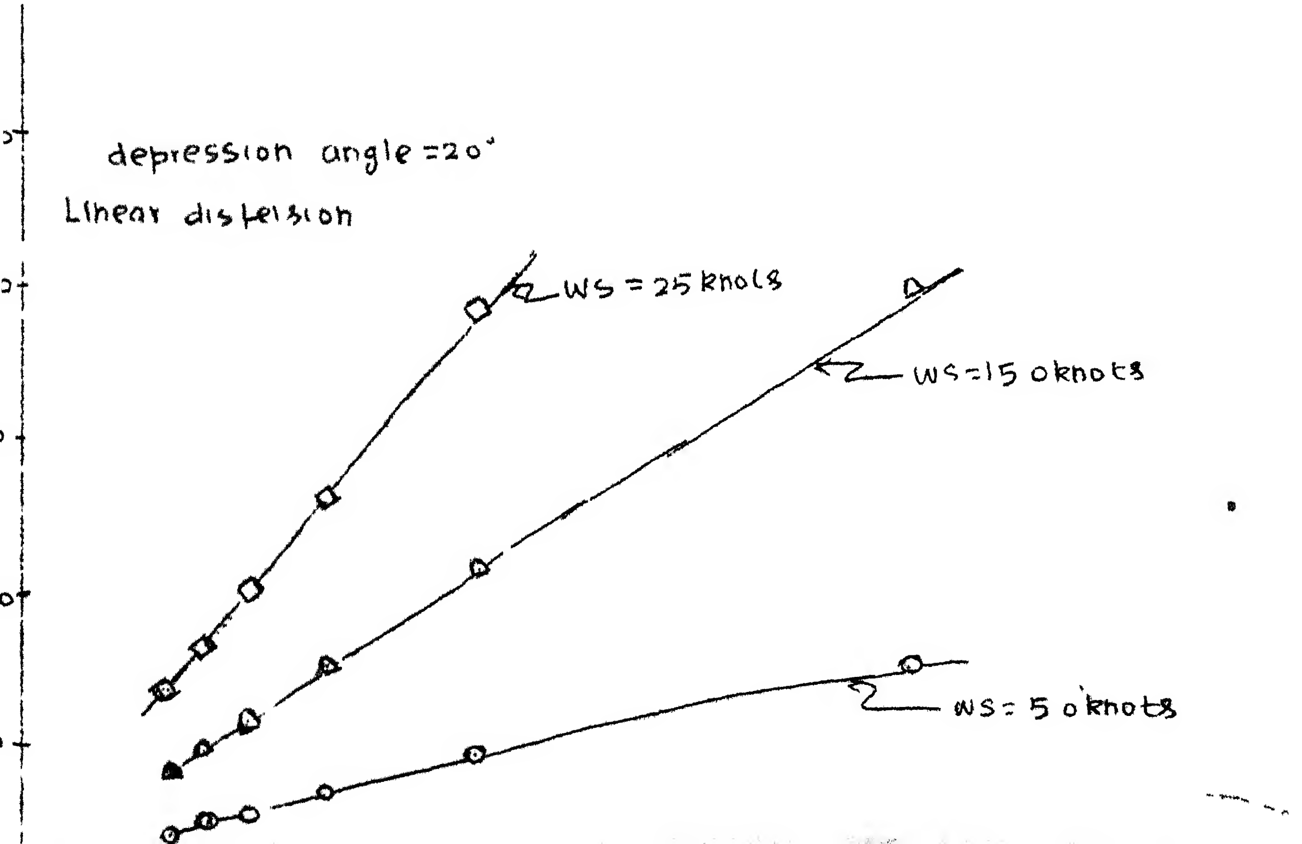


Fig 6 2 5 WIND VELOCITY (knots) →



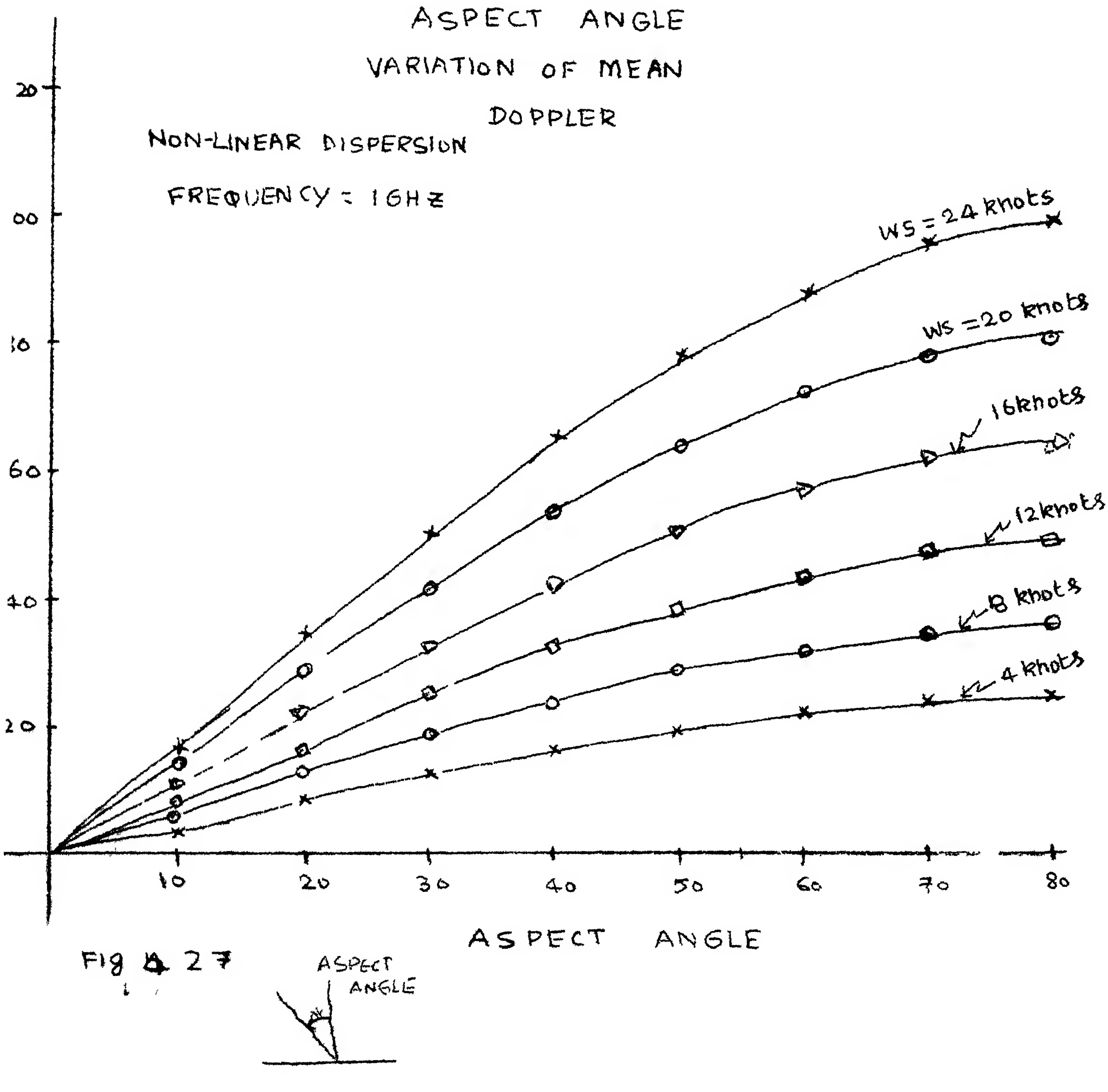
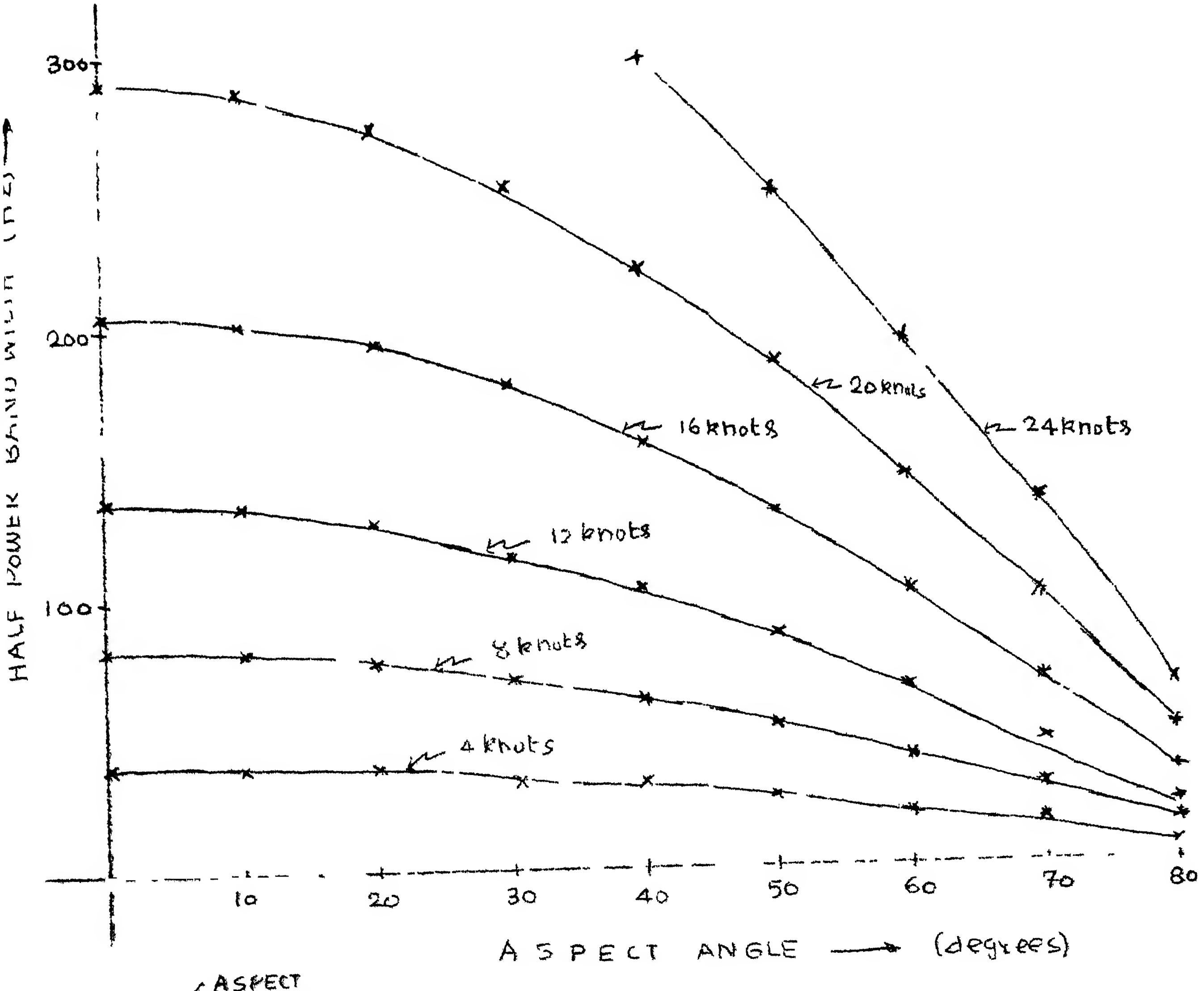

ASPECT ANGLE
VARIATION OF MEAN
DOPPLER
NON-LINEAR DISPERSION
FREQUENCY = 1GHZ
WS = 24 knots
WS = 20 knots
16knots
12knots
8 knots
4 knots
20
00
60
40
20
10
20
30
40
50
60
70
80
ASPECT ANGLE
ASPECT
ANGLE

Fig 4.27

ASPECT ANGLE VARIATION OF
HALF-POWER BANDWIDTH
NON-LINEAR DISPERSION



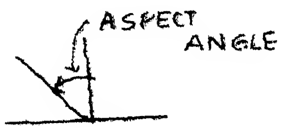


FIG 4 2 8

APPENDIX - A

The eigen value problem $S\underline{e} = t\underline{e}^*$ ....(A.1)

can be solved as follows.

Consider the conventional eigen vectors and eigen-values of the problem $S^+S\underline{x} = \lambda \underline{x}$ ....(A.2)

since S is a complex, symmetric matrix, S^+S is a Hermitian matrix. Hence, (1) The eigen-vectors of S^+S matrix are orthogonal. (2) The eigen-values of S^+S are real.

Let us assume that $S\underline{x} = \underline{e}^*$ ....(A.3)

Substituting (A.3) in (A.2),

$$S^+\underline{e}^* = S^*\underline{e}^* = \lambda \underline{x}$$

since $S^*\underline{e}^* = t^*\underline{e}$,

$$t^*\underline{e} = \lambda \underline{x} \quad ....(A.4)$$

multiplying both sides by S,

$$t^*S\underline{e} = S\underline{x}$$

$$|t|^2 e^* = e^*$$

$$\lambda = |t|^2 \quad ....(A.5)$$

substituting in (A.4), we obtain,

$$t^*\underline{e} = |t|^2 \underline{x}$$

$$\underline{e} = t\underline{x} \quad .....(A.6)$$

Thus the relations (A.3) and (A.6) express, the eigen-values of the problem $Se=te^*$ om terms of conventional eigen-values and eigen-vectors of the problem $S^+S\underline{x}=\lambda$

CHAPTER 5

CONCLUSION

5.1 CONCLUSION:

In this thesis an attempt has been made to model Radar-clutter targets in terms of their polarization properties of the scattered returns. The scattered waves are decomposed into two parts - a completely coherent part corresponding to deterministic part of the target and a incoherant part corresponding to a noise target. This decomposition is carried out by the use of Stokes Reflection Matrix associated with the target. The Target Scattering Matrix corresponding to the deterministic part of the target is obtained and a set of five phenomenological parameters associated with the target are also calculated.

In order to illustrate above ideas, for a rough surface treated as a random rough surface the mean and variance of Target Scattering Matrix associated it are computed. These values are utilized to obtain Stokes Reflection Matrix. The expressions for mean and variance of the TSM are modified to include the temporal variations of the surface. The expressions derived are applied to an asphalt road and a sea-surface and the 'Five phenomenological parameters associated with these clutter-targets are obtained.

The variation of these parameters with radar and surface parameters are also investigated.

5.2 SUGGESTIONS FOR FURTHER WORK:

In the above model, we evaluated the integral given in equation (3.1.7) making a number of approximations. A major approximation has been to replace the angles between local planes and incident planes at different scattering points in the illuminated area by their average value. A more accurate and realistic approach would be to incorporate the angle-height dependence in evaluating the integral.

In computing the Stokes Reflection Matrix, we have taken into account only the surface scattering component of the scattered field. This obviously restricts the applicability of the model to those situations in which surface scattering is predominant or volume scattering effects are negligible. But clutter-targets like vegetation, sea-ince and show, where volume scattering effects are not negligible, the Stokes Reflection Matrix must be computed by taking in to account the volume scattering effects [16,27].

[10] Bahar E. 'Full wave approach to scattering from rough surfaces, IEEE - trans - AP - 1981. pp 443-454.

[11] Barrick D.E., 'Rough surfaces' chap 9 in 'Radar cross section handbook', ed.by G.T.Ruck, Plenum.1970.

[12] Krauss and Carvar, 'Electromagnetics', second edition, International Students edition 1973, chap 11.

[13] Welf E., 'Coherance properties of partially polarized EM Radiation', Nuovo cemento, ser 10, pp.1165-1181, 1959.

[14] Perrin F., 'Polarization of light scattered by isotropic opalescent media', J. of chem.Phys (10) pp. 415-427.

[15] Hurwitz, 'The statistical properties of unpolarized light', J.of Opt. Soc. Am. (85) pp. 525-531, 1942.

The above refs. [13-15] are available as Reprint papers in 'Polarized Light' ed.by W.Swindell, Halstead press.

[16] Ishimaru A. 'Wave propagation and scattering in random media' - Academic Press - 1978 vol. I and II chap. 21.

[17] Wright J.W., 'A new model for sea clutter', IEEE-AP-1968 pp. 217-221.

[18] Valenzuela G.R., 'Theories for interaction of electromagnetic and oceanic waves', Beundry-layer Metreology-1978, pp. 61-84.

[19] Kinsman, 'Wind Waves', Prentice-Hall - 1965.

[20] Lane J.A. and Saxton, 'Electrical properties of sea-water', Wireless Engineer, vol.29, Oct.1952 p.269.

[21] Long M.W., 'Radar Reflectivity on Land and Sea', Heath - 1975.

[22] Ulaby F.T., Moore R.K., Fung A.K., 'Microwave remote sensing - active and passive', vol.2, Addison Wesley - 1982.

[23] R.L. Cosgriff et.al, 'Terrain handbook - II', The Ohio State University, May 1960, Reprint in 'Radars Vol.5' edited by D.Barton, Artech House Inc., 1975.

[24] Kalmykov A.I. and Pustovoytenko V.V., 'On polarization features of radio signals scattered from sea surface at small grazing angles', J.of Geophysical Research, vol.81 (12) April 1976.

[25] Nathanson F.E., 'Radar Design principles - signal processing and the environment', McGraw Hill, 1969.

[26] Smith B.G. 'Geometrical shadowing of a Random Rough Surface' IEEE-Trans-AP-15 - pp 668-671,1967.

[27] Fung A.K., EOM H.J. 'Scattering from random layer with applications to snow, vegetation and seaice' Proc IEE (London) - Part F- pp Dec 1983.

```
[illegible]
[illegible] ... [illegible] the surface is modelled as a [illegible] gaussian
[illegible] ... [illegible] deviation [illegible]
[illegible] surface correlation length [illegible]
[illegible] constant [illegible]
**************************************************************************

**************************************************************************
[illegible]
```

```
[illegible]

[illegible]
[illegible]

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
```

[illegible]

```
X21=(D+SQRT(R))/(2*Y1)
X22=(D-SQRT(R))/(2*Y1)
Y21=(-D-SQRT(R))/(2*X1)
Y22=(-D+SQRT(R))/(2*X1)
YY=Y1*Y21; YYY=Y1*Y22
IF(YY.LE.0) GOTO 17
Y2=Y21; GOTO 19
Y2=Y22
IF(X21.LE.0) GOTO 21
X2=X21; GOTO 22
X2=X22
PI=3.14159265
X1PY1=X1+Y1; X2PY2=X2+Y2
Z2=(M14*X2PY2+M32*X1PY1)/(X2PY2*X2PY2+X1PY1*X1PY1)
Z1=(M14-(Z2*X2PY2))/X1PY1
S11=CMPLX(X1,X2)
S12=CMPLX(Z1,Z2)
S22=CMPLX(Y1,Y2)
PRINT 198
FORMAT(/,10X,'SCATTERING MATRIX ELEMENTS ARE')
PRINT *,S11,S12,S22
CALL EIGEN(X1,X2,Y1,Y2,Z1,Z2,T1,T2)
T1A=CABS(T1);T2A=CABS(T2)
T1P=ATAN(AIMAG(T1)/REAL(T1))
T2P=ATAN(AIMAG(T2)/REAL(T2))
PRINT [illegible]
FORMAT(10X,'CHECK VALUES ARE')
PRINT *,T1P,T2P
IF(T1A.GT.T2A) GOTO 123
TMAG=T2A
NUR=(T2P-T1P)/4
GAMMA=ATAN(T1A/T2A)*180/PI; GOTO 124
GAMMA=ATAN(T2A/T1A)*180/PI
NUR=(T1P-T2P)/4
TMAG=T1A
PRINT 127,TMAG
FORMAT(/,10X,'TARGET MAGNITUDE IS=',F10.5)
PRINT 125,GAMMA
FORMAT(/,10X,'CHARACTERISTIC ANGLE IS='F6.3)
NU=NUR*180/PI
PRINT 129,NU
FORMAT(/,10X,'TARGET SKIP ANGLE IS=',F5.3)
RETURN
END
SUBROUTINE EIGEN(X1,X2,Y1,Y2,Z1,Z2,T1,T2)
REAL AR(3,3),AI(3,3),WR(3),WI(3),VR(3,3),VI(3,3)
INTEGER I,J,N,IAR,IAI,IVR,IVI,IFAIL,INTGER(5)
COMPLEX E1,E2,EV1,EV2,T(3),T1,T2
X11=X1*X1+X2*X2+Z1*Z1+Z2*Z2
Y11=Y1*Y1+Y2*Y2+Z1*Z1+Z2*Z2
```

```
Z11=X1*Z1+X2*Z2+Y1*Z1+Y2*Z2
Z22=X1*Z2-X2*Z1+Y2*Z1-Y1*Z2
N=2; AR(1,1)=X11; AR(1,2)=Z11; AR(2,1)=Z11; AR(2,2)=X11
AI(1,1)=0.0; AI(1,2)=Z22; AI(2,1)=Z22; AI(2,2)=0.0
IAR=3; IAI=3;IVR=3;IVI=3;IFAIL=0
CALL F02AKF(AR,IAR,AI,IAI,N,WR,WI,VR,IVR,VI,IVI,INTGER,IFAIL)
IF(IFAIL.EQ.0) GOTO 85
WRITE 40,IFAIL
RETURN
WRITE 70
EG1=WR(1); EG2=WI(1); EG3=WR(2); EG4=WI(2)
FORMAT(10X,'EIGEN VALUES')
WRITE 60, (WR(I),WI(I),I=1,N)
WRITE 100
FORMAT(10X,'EIGEN VECTORS')
DO 90 I=1,N
WRITE 60 ,(VR(I,J),VI(I,J),J=1,N)
CONTINUE
I=0
DO 200 I=1,N
A1=VR(I,1); B1=VR(I,2)
A2=-VI(I,1); B2=-VI(I,2)
E11=A1*X1-A2*X2+B1*Z1-B2*Z2
E12=A1*X2+A2*X1+B2*Z1+B1*Z2
E21=A1*Z1-A2*Z2+B1*Y1-B2*Y2
E22=A1*Z2+A2*Z1+B1*Y2+B2*Y1
E1=CMPLX(E11,-E12)
E2=CMPLX(E21,-E22)
PRINT 205,I
FORMAT(/,10X,'MODIFIED EIGEN VECTOR',I2,2X,'IS')
PRINT *,E1,E2
EV1=CMPLX(A1,-A2)
EV2=CMPLX(B1,-B2)
IF(EV1.EQ.(0.0,0.0)) GOTO 210
T(I)=E1/EV1
GOTO 220
T(I)=E2/EV2
PRINT 230,I
FORMAT(/,10X,'MODIFIED EIGEN VALUE',I2)
PRINT *,T(I)
CONTINUE
T1=T(1);T2=T(2)

RETURN
FORMAT(25H0 ERROR IN F02AKF IFAIL= ,I2)
FORMAT(1H,2(1H(,F8.6,1H,F8.5,1H),1X))
END
```